A Model for Development of Optimized Feeder Routes and Coordinated Schedules
– A Genetic Algorithms Approach

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Abstract

Many attempts have been made to solve bus route network design problems by splitting it in two stages,
one for routing and the other for scheduling. Some researchers have made attempts to solve network
design problems using non traditional optimization techniques also, but not much has been done on
modeling coordinated operations involving transfers from one mode to another. In this research, feeder
routes and frequencies leading to schedule coordination of feeder buses with main transit are developed
simultaneously using Genetic Algorithms. The coordinated schedules of feeder buses are determined for
the existing given schedules of main transit. Thus the developed feeder routes and schedules are
complementary to each other. As a case study the Dun Laoghaire DART (Dublin Area Rapid Transit)
(heavy rail suburban service) station of Dublin in Ireland is selected. Finally the outcome of the research
is a generated feeder route network for feeder buses and coordinated schedules of feeder buses for the
existing schedules of DART at the selected station. The results of the proposed model indicate improved
load factors on developed routes and also the overall load factor is also improved considerably as
compared to authors’ earlier model.

Key Words: Routing and scheduling, Genetic Algorithms, optimisation, coordinated operations, public
transportation, intermodal coordination.

1. Introduction

The efficiency of the public transport system in any city can be enhanced by integrating its components.
In metropolitan cities of developed and developing countries suburban railway and public buses are major
public transport facilities. In most of the cases each public transport facility is planned and designed
independently. The independent operations develop competition among public transport modes leading to
duplication of services and uneconomical operations. Lack of integration increases the transfer and journey times. The higher journey and transfer times cause dissatisfaction to commuters and they lose patronage towards public transport facility. Optimized feeder routes and coordinated schedules reduce the overall journey time of commuters. The purpose of feeder routes is to connect all the destinations for which demand generates from railway station. However the feeder buses can be made to stop on locations also on the route if there are demands.

1.1 Literature review and objective of study

In most previous studies, routes and schedules are developed independently so as to avoid complications and computational burden. Most of the researchers have developed routes from a given initial skeleton using a heuristic approach. Lampkin and Saalmans (1967), Silman et al. (1974), Dubois et al. (1979), Hsu and Surti (1976), Dhingra (1980), Mandl (1980), Baaj and Mahamassani (1990 and 1995), developed bus routes using a heuristic approach by insertion of nodes in the base network. The heuristic approach may not always guarantee an optimal route structure. So far, very limited studies are made for generation of feeder routes and schedules for coordinated operations (Shrivastava and Dhingra, 2000). However, Wirasinghe (1980), Geok and Perl (1988) attempted routing and scheduling problems for coordinated operations using analytical models. They had considered a highway grid which is assumed to be rectangular and parallel to a single railway line which may not always be true in practice. They had made an attempt to describe a complex transit system by approximate analytical models. Thus most of the studies on coordination of modes are limited to analytical modeling without considering a real life network. Shrivastava and Dhingra (2001) developed feeder routes using a heuristic approach. These routes were developed independently without considering the impact of the schedules of feeder buses.

Large numbers of studies are available for problems related to scheduling of public transport modes. Optimization, simulation and expert system approaches are adopted for many scheduling problems with single and multiple objectives (Shrivastava and Dhingra, 2000). Limited studies have been made to obtain optimal schedules for transit networks with transfer time consideration which is a prime concern for schedule coordination problems. Attempts have been made using computer simulation (Rapp and Gehner, 1976) and using a combination of optimization and simulation procedures (Bookbinder and
However development of such an optimal schedule is an extremely difficult task even for a small network (Kikuchi and Parmeswaran, 1993). The difficulty arises because of the large number of variables and constraints, the discrete nature of variables, and the non-linearity involved in the objective function and the constraints (Chakroborty et al., 1995). In view of this, techniques like fuzzy logic have been tried for such problems (Kikuchi and Parmeswaran, 1993). Shrivastava and Dhingra (2002) successfully attempted schedule coordination problem using Genetic Algorithms (GAs).

A combined routing and scheduling problem was attempted by Hasselstorm (1981) and Marwah et al (1984). They used a complex two-level optimization model, which first reduces the network by eliminating links and then assigning frequencies using a linear programming model which maximizes the number of transfers saved by changing from a link network to a public transport network. Tom and Mohan (2003) attempted transit route network design using frequency coded Genetic Algorithms for an urban bus system. Shrivastava and O’Mahony (2005) developed a model for development of feeder routes and schedule coordination. In this study, feeder routes and coordinated schedules for feeder buses were developed in two stages using routing and scheduling sub models.

Thus, the literature review reveals that many studies are available for development of routes, schedules, combined routing and scheduling leading to design of a network. Limited studies are available for operational integration of public transport modes involving development of feeder routes and schedule coordination with transfer time consideration between two modes.

The objective of this research is development of a model for generation of feeder routes and coordinated schedules simultaneously. In actual practice routes and schedules go hand in hand. If routing and scheduling go together the generated routes support the defined schedules. In view of this, in this study optimised feeder routes and coordinated schedules are developed together using GAs. As a case study, Dun Laoghaire DART (Dublin Area Rapid Transit) station in Dublin, Ireland is selected.

1.2 Details of study area

The Dublin Area Rapid Transit (DART) is a suburban railway system in Dublin, running basically along the coastline of Dublin Bay from Greystones to Howth and Malahide. There are 32 stations on the existing DART line. Figure 1 indicates the existing DART line along with various DART stations. Dun Laoghaire is one of the prominent DART station where after arriving by DART large number of commuters travel
by different modes to local destinations. In the absence of an appropriate feeder route system and lack of coordination between DART and the buses, commuters have to wait and travel longer to reach their destinations.

Considerable movement of commuters takes place towards Sallynoggin, Monkstown, Deans Grange, Stillorgan and Loughlinstown areas from the DART station. Currently route numbers 75, 111, 59, 45A, 46A and 46X originate from the station whereas route numbers 7 and 7A originate elsewhere and pass through the station. Some of the existing routes that originate at a station pass through some of the locations for which demand does not originate from the station as indicated in our typical traffic survey. Bus route 75 passes through Stillorgan, Leopardstown, Sandyford, Ballinteer, Oldbawn and Tallaght. Our survey shows that typically the demand for last three destinations is nil from the station. Moreover the length of this route is very long at more than one hour. Generally, longer routes pose problems in maintaining schedules. Similarly bus route 46A goes to the city centre thus duplicating the services of the DART towards the city centre. Bus route 45A goes to Bray which is parallel to the DART line. Route 59 passes through Dalkey and Killiney. Route 111 also passes through Dalkey and goes to Loughlinstown. Routes 59 and 111 could be clubbed together and a single feeder route would serve the purpose.

It can be concluded that the existing routes do not serve the purpose of feeder routes and they also lead to duplication of services. Since the existing routes do not serve the purpose of feeder routes, the waiting time of commuters at Dun Laoghaire DART station for these routes is found to be about 20 minutes and some times even more during the morning peak hour. Some commuters have to walk more than 10 minutes to reach their destinations.

2. Data Collection and Analysis

Typical traffic surveys were conducted during the morning peak period i.e. 7 to 9 a.m. on April 28, 2004. It was observed that the maximum number of commuters travel during 8 to 9 a.m. Therefore this time period is identified as the peak hour. It has been confirmed during traffic surveys that after 9 a.m. commuter traffic starts decreasing and becomes much less after 9.30 a.m. onwards.

During the traffic surveys, commuters exiting the DART station were counted manually. Typical commuter counts revealed that between 8 and 9 a.m., 1293 commuters exit from the DART station.
Traffic surveyors conducted sample interviews of commuters leaving the DART station. Between 8 and 9 a.m. 300 commuters were interviewed thus making a sample size above 20%. Enquiries were made regarding their destinations, mode of transport and travel time to their destinations from the DART station.

The actual number of commuters using cars / taxis was counted. The expected improved integrated scenario in terms of reduction in transfer time (less than 5 to 10 minutes) from DARTs to feeder buses was presented to these commuters. The percentage of these commuters willing to shift to feeder buses was determined based on their responses. Since a small percentage of DART commuters were using taxis / cars for their onward journey from DART station, it was possible to get exact number of such commuters. Similar methodology was successfully implemented by Shrivastava and Dhingra (2001) in the city of Mumbai, India. The percentages of commuters shifting to public buses were added to those already using public buses. Destination wise summation gave potential demand matrix. It was found that 40% of commuters have their working places very near to the DART station and they have to walk even less than 5 minutes. These commuters were not interested in shifting to public buses even if they are well coordinated with DART services.

Fifteen potential destinations were identified for the analysis. Connectivity details and distances between destinations were obtained from a Dublin Street map and a travel distance matrix of size $16 \times 16$ which includes the DART station and 15 other destinations as indicated in Table 1 was developed. The potential demand matrix, travel time matrix and link details were used for analysis.

It was also observed during traffic surveys that in the morning peak period the trains towards city centre (north bound trains or $n^{th}$ bound) contribute about 30% passengers; the remaining 70% were by trains from city centre (south bound trains or $s^{th}$ bound). There were nine north bound and eight south bound trains during the peak hour of 8 to 9 a.m. The schedule coordination for feeder buses is attempted for these trains during the indicated peak hour.

3. Model for Development of Optimized Feeder Routes and Coordinated Schedules

The modelling exercise mainly attempts to address the problem of the selection of feeder routes and coordinated schedules simultaneously. This exercise helps in finding an optimal fleet size for an optimally designed feeder route network. The feeder route network is to be developed for commuters originating from the DART station and subjected to one transfer i.e. from DARTs to feeder buses. Since
the commuters are already subjected to one transfer, feeder route network is designed without additional transfer which leads more discomfort to passengers.

The objective function is adopted as minimization of the user’s and operator’s costs. The user’s cost is the summation of in-vehicle time cost and the transfer time cost between DARTs and buses. The operator’s cost is associated with running cost (vehicle operation cost) of buses. Constraints are related to load factor, fleet size and unsatisfied demand. The selected set of k – paths (length \( l_j \)) for each potential destination and set of frequencies on these paths are the decision variables. The objective function and various constraints are given as below. It can be seen that vehicle operation cost and transfer time between DARTs & coordinating buses depends on selected set of frequencies \((f_j)\) of buses on different paths. Travel time cost will depend on selected set of lengths of k-paths. Similarly all the constraints are function of frequencies of buses \((f_j)\). Fleet size constraint is a function of both decision variables.

3.1 Objective function, constraints and penalties

**Objective Function:**

Minimize Total cost, \( Z = (\text{Transfer time cost between DARTs and Coordinating buses} + \text{travel time cost in buses on selected routes} + \text{Vehicle operation cost of Dublin buses}) \)

Mathematically it can be expressed as,

Minimize \( Z = \)

\[
C_1 \left( \sum_j \sum_u \sum_l \text{pass}^u_j (\text{bus}_l^1 - \text{dart}^u) \delta_j^{u1} + \sum_j \sum_u \sum_l \text{pass}^v_j (\text{bus}_l^1 - \text{dart}^v) \delta_j^{v1} \right) + \\
C_2 \left\{ \sum_j \text{pass}_j \times t_{\text{inv},j} \right\} + C_3 \left\{ \sum f_j T_l \right\}
\]

**Constraints**

1. \( \frac{Q_{j,\max}}{f_j \times T \times CAP} \leq L_{\max} \) (load factor should be less than its maximum value)

2. \( \frac{Q_{j,\max}}{f_j \times T \times CAP} \geq L_{\min} \) (load factor should be more than its minimum value)
3. \[ \sum_{j \in SR} \frac{\sum_{k \in SR} \left( \frac{f_j \times T \times (RT_j)}{TP} \right)}{NB_j} \leq W \text{ (for all } j \leq SR) \text{ (Fleet size should be less than adopted value)} \]

4. \[ \sum_{j} d_{unsat} = 0 \text{ (Unsatisfied demand should be as less as possible)} \]

Where,

\( j \) = Number of routes available at each stations (as per number of potential selected destinations)

\( l \) = Number of buses available for \( u^{th} \) north bound DART and \( v^{th} \) south bound DART

\( VOC \) = Vehicle operating cost for Dublin buses

\( C_1 \) = Cost of transfer time in Euro /minute, 11.32 cents/minute for the case study, (Steer Davies, 1994).

\( C_2 \) = Cost of in vehicle time in Dublin buses, 0.076 cents/minute for the case study (Steer Davies, 1994)

\( C_3 \) = Cost of operation of bus per Km., € 3.66 for Dublin buses for the case study, (Scott Wilson, 2000).

\( pass_{u} \) = Passengers transferring from \( u^{th} \) north bound DART to \( j^{th} \) route.

\( pass_{v} \) = Passengers transferring from \( v^{th} \) south bound DART to \( j^{th} \) route.

\( pass_j \) = Total number of passengers transferring to \( j^{th} \) route

\( bus_j \) = Departure of \( l^{th} \) bus on \( j^{th} \) route

\( dart_u \) = Arrival of \( u^{th} \) north bound DART

\( dart_v \) = Arrival of \( v^{th} \) south bound DART

\( \delta_{j}^{u,l} \) = is a term which shows whether the transfer of passengers is possible or not. It attains a value of one if the transfer is from a \( u^{th} \) north bound DART to \( l^{th} \) bus on \( j^{th} \) route at the DART station is feasible; otherwise it attains a zero value.

\( \delta_{j}^{v,l} \) = is also a term which shows whether transfer of passengers is possible or not. It attains a value of one if the transfer is from \( v^{th} \) south bound DART to \( l^{th} \) bus on \( j^{th} \) route at DART station is feasible otherwise it attains a zero value.

\( t_{inv,j} \) = In-vehicle time in bus on \( j^{th} \) route

\( f_j \) = Frequency of buses on \( j^{th} \) route in terms of number of bus trips per hour.

\( l_j \) = length of \( j^{th} \) route in kilometers

\( T \) = Time period, hours

\( Q_{j,max} \) = Maximum number of passengers on \( j^{th} \) route for the given time period.
CAP = Seating capacity of bus, for Dublin buses it is taken as 74 (Scott Wilson, 2000)

$L_{\text{max}}$ = Maximum load factor, it is adopted as 1.2 for the case study (Scott Wilson, 2000)

$L_{\text{min}}$ = Minimum load factor, adopted as 1 for the case study

d$_{\text{unsat}}$ = Unsatisfied demand

SR = Set of Routes

$NB_j$ = Number of buses required in any route ‘j’

$(RT)_j = \text{Round trip time of the bus on } j^{\text{th}} \text{ route in minutes} = 2 \times t_j \text{ (in minutes)} + \text{layover time (5 minutes for the case study)}$

$t_j = \text{Total travel time on route in minutes including stopping times (Journey time)}$

TP = Time Period in minutes

$N_j = \text{Number of trips per hour multiplied by the time period in hours under consideration ( } f_j \times T )$

$W = \text{Maximum number of available buses}$

The first two terms of objective function indicate the user’s cost and the third term the operator’s cost. The user’s cost is taken as the summation of costs associated with transferring from DART services (both north and south bound) to coordinating buses (first term) and the cost of travel time while on buses (second term). The operator cost is in terms of vehicle operating cost, which is proportional to the distance traveled by buses (third term). Constants $C_1$, $C_2$ and $C_3$ are used to convert each term of objective function into monetary unit of Euro (€). As discussed above the vehicle operation cost is taken as cost of bus operation per kilometer ($C_3$). The value of $C_3$ could be determined easily and accurately from the literature (Scott Wilson, 2000). The effect of congestion cost is also indirectly incorporated in the objective function by adopting an average speed of 15 km per hour (Scott Wilson, 2000). Thus the in-vehicle time term depends on lengths and average speed on selected routes.

It was ascertained that only majority of DART commuters seek transfer at the selected bus stops at the DART station. Thus transfer time is predominant in the case study. Waiting time will be an important term if apart from DART commuters substantial numbers of commuters originate from bus stops (at
DART station). Since percentage of such commuters were very less as compared to DART commuters the effect of waiting time is not considered.

The first and second constraints ensure that the load factor lies within a maximum and a minimum value. If the load factor is less than a maximum value then the crowding level will be less and a better level of service will prevail. The level of service should not be less than a minimum value so as to ascertain availability of a certain minimum number of passengers for economical operations. The maximum load factor is the ratio of crush capacity to normal capacity of Dublin buses. The crush capacity is taken as 88 and normal capacity is 74; thus the maximum load factor which is the ratio of the two capacities is taken as 1.2 (Scott Wilson, 2000). The value of minimum load factor is taken as 1.00. The third constraint is associated with fleet size. This constraint puts the upper limit on the maximum number of available buses for operation. The fleet size of ‘25’ is decided based on the existing requirement of buses (on route numbers 75, 111,59,45A) which originate from DART station and pass through most of the destinations as identified in our typical traffic survey. Later, after analysis, it was found that the adopted value of 25 is more than the requirement on selected feeder routes. However the fleet size is a constraint which depends on availability of buses. It was also observed in our typical traffic survey that the demand for feeder buses gradually increases and reaches a maximum during the peak hour. During the peak hour, the number of commuters transferring from DARTs to buses remains stable. After the peak hour, the demand gradually decreases for feeder buses decreases. This assumption is nearly justified because during the peak hour the demand for feeder buses remains stable, uniform and at a maximum. However since the demand matrix is a variable, the appropriate demand can be taken for the analysis during different periods of the day. The fourth constraint ensures that the maximum demand is satisfied and that the maximum numbers of commuters get coordinating buses during the period of analysis (Shrivastava et al, 2002).

The objective function and constraints in the study pose a constraint optimization problem; therefore a penalty method is adopted for optimization (Deb, 1995). If any constraint is violated, an appropriate penalty is calculated and added to the objective function and thus a penalized objective function is found. Using the Genetic Algorithms the minimum value of the penalized objective function is
determined (best fitness). The routes and frequencies corresponding to this minimum value are optimal. These optimal frequencies are used to determine coordinated schedules of feeder buses.

Fitness function = Minimize (Objective Function + Penalties 1 to 4)

Penalty 1: if load factor is more than a maximum value (1.2 for the case study)

Penalty 2: if load factor is less than minimum value (1 in this case study)

Penalty 3: if fleet size exceeds a minimum value (25 for the case study)

Penalty 4: if some demand remains unsatisfied

The set of penalties are decided so as to keep the load factors of buses within the prescribed ranges, to keep the percentage of unsatisfied demand as low as possible and the fleet size less than the specified value. These penalties are decided after several trials of GA using different sets of values. The GA parameters are tuned for the objective function and constraints before deciding on the penalties. It is also observed in our boarding/alighting surveys that it takes about five minutes for commuters to reach the bus stops located outside the station after arriving on a DART. Thus the minimum desirable transfer time required from DART to buses is taken as five minutes. Hence in the analysis transfer from a DART to a bus is taken as feasible only if the bus departs after five minutes of scheduled arrival of the DART.

3.2 Steps involved for development of model

The proposed overall methodology is depicted in Figure 2 for developing feeder routes and coordinated schedules simultaneously.

1) The potential demand matrix is developed with the DART station as origin and the destinations as identified in the surveys.

2) The travel distance matrix and connectivity details among various nodes were obtained from the map.

   The travel distance matrix was converted into a travel time matrix in ‘minutes’ using an average speed of 15 km / hr to address the existing congestion level and road geometrics of the influence area (Scott Wilson, 2000).

3) The other parameters like capacity of buses, minimum and maximum load factors, maximum fleet size, minimum and maximum acceptable lengths of feeder routes, minimum frequency per hour, multiplying constants to convert user and operator costs in to monetary value of ‘euro’ are selected.
(4) Keeping the DART station as the origin, the potential destination nodes are identified on the basis of potential demand and their location. Thus the nodes which are very close to the DART station are not selected as potential destinations in spite of having high demands. Selection of such nodes leads to development of very small routes mushrooming near to the station (Shrivastava and Dhingra, 2001) and a higher percentage of demand remains unsatisfied due to remote destinations. In view of this, priority for selection of potential destinations is given to the nodes away from the origin and well scattered in the influence area of the DART station with a preference to higher demand nodes (if they are available). Generation of feeder routes and corresponding frequencies is repeated till selected set of potential destinations satisfies entire demand of the influence area.

(5) Using the K-Path Algorithm, K- shortest paths were developed between DART station and identified potential destinations (Eppstein, 1994). The genetic algorithm selects the best k – path out of a total developed for a pair of origin and destination. Thus the k – paths are not predetermined. They are in fact selected by the genetic algorithms along with a set of bus frequencies. The details of selection of routes and frequencies by using genetic algorithms are discussed in the section on implementation of genetic algorithms’. The value of ‘K’ is adopted as five for the case study. In the influence area, five nodes which were away from the DART station, well scattered and covering almost entire influence area were selected as potential destinations. The number of destinations available was sixteen and five potential destination were identified.’. The remaining nodes for development of the routes were only 11. Therefore, five k – paths for each destination would be enough. However if the number of nodes are more, then more than a number of k – paths can be developed.

(6) The programme to calculate the penalised objective function (summation of objective function and penalties due to violation of constraints) is developed in a ‘C++’ environment. The developed ‘k’ paths are used with the objective function program. The provision is made so that each alternative path out of five ‘k’ paths for each potential destination is selected with random frequencies generated by GA. The alternative paths and frequencies corresponding to the minimum penalized objective function are selected as optimized feeder routes and frequencies. Assignment of trips is carried out by exponential function of frequency and distance to make the model more universal (Gundaliya et al, 2000). The exponential function is used to distribute the trips, as per bus frequencies and length of
routes. This function has been adopted because choice of routes depends not only on frequencies of buses but also on length of alternate routes.

The developed objective function is used with LibGA software (Lance Chambers, 1995) in a Linux environment. GA parameters are tuned and penalties for unsatisfied demand, minimum & maximum load factors and maximum fleet size are decided. Results obtained by GA are very sensitive to penalties. Slight changes in penalties show wide variation in results (Shrivastava and O’Mahony, 2005). In view of this, the final values are decided after several trials using different sets of values for penalties. The minimum value of the penalised objective function is determined and feeder routes and frequencies corresponding to this minimum value are the optimal feeder routes and frequencies. Coordinated schedules of feeder buses are derived from these optimal frequencies.

Required number of buses is calculated from optimal frequencies as obtained above (for the case study typical calculation for number of buses is made assuming same optimal frequencies after selected peak hour, in fact requirement of buses will decrease in off peak hours and thus frequency will get affected)

3.3 Application of GA for Model Development

The importance of this type of problem which is attempted in this research had been highlighted by Chakroborthy et al (1995) previously. Even after linearizing the problem, complexity remains very large. The benefit obtained through linearization is offset by the increase in number of variables and constraints. In general, the number of variables and constraints required are of the order of \(O(r^2n^2)\), where ‘\(r\)’ is the number of routes through the transfer station and ‘\(n\)’ is the number of buses/trains on any of the routes. Chakroborthy et al (1995) attempted to solve the linearized formulation of similar problem, but the algorithm failed to converge to any solution. Based on this experience and that reported for a similar problem elsewhere (Kikuchi and Parmeswaran 1993), it was decided to use a robust optimization technique like Genetic Algorithms rather than classical optimization methods.

3.3.1 Overview of GAs

Genetic Algorithms (GAs) proposed by Holland (1975) are based on exhaustive and random search techniques, found to be robust for optimizing non-linear and non-convex functions. In the application of
Genetic Algorithm the decision variables are usually mapped and represented by a string (Chromosome) of binary alphabets (genes). For problems with more than one decision variable, a sub-string usually represents each variable. All sub-strings are then concatenated together to form a bigger string.

The operation of GAs begins with population of random strings representing design of decision variables. Thereafter, each string is evaluated to find the fitness value. The population is then operated by three main operators: reproduction, crossover and mutation to create a new population of points. ‘Reproduction’ selects good string in population, ‘crossover’ operator exchanges the information among strings of mating pool and ‘mutation’ makes the local search around current solution. The new population is further evaluated and tested for termination. If the termination criterion is not met, the population is iteratively operated by above three operators and evaluated. This procedure is continued until the termination criterion is met. One cycle of these operations and subsequent evaluation procedure is known as a ‘generation’. Since the research problem falls in the category of constrained optimization problem, the penalty method which is well suited for such problems is used (Deb, 1995). The Genetic Algorithms has following advantages over conventional optimization techniques.

(i) GAs work with a string coding of variables instead of the variables. The advantage of working with a coding of variables is that coding descretizes the search spaces, even though the function may be continuous. On the other hand, since GAs require only function values at various discrete points, a discrete or discontinuous function can be handled with no extra burden.

(ii) Since no gradient information is needed in GAs, they can also be applied to non-differentiable functions. This makes GAs robust in the sense that they can be applied to a wide variety of problems. Moreover, GAs exploits coding similarities to make a faster and parallel search.

(iii) The greatest advantage of GAs over many traditional optimization methods is that GAs work with a population of points instead of a single point. Because there are more than one string being processed simultaneously, it increases the possibility of obtaining global optimum solution even in ill-behaved functions (Goldberg 1989). Even though some traditional algorithms are population based, like Box’s evolutionary optimization and complex search methods, those methods do not use previously obtained information efficiently as GA does.
GAs use probabilistic transition rules instead of fixed rules. In early GA iterations, this randomness in GA operators makes the search unbiased towards any particular region in the search space. As a result of not making a hasty wrong decision, it also affects a directed search later in the optimization process. Uses of stochastic transition rules also increase the chances of recovering from a mistake.

3.3.2 Implementation of Genetic Algorithms

Since the objective of the problem is to develop feeder routes and coordinated schedules simultaneously, the information about routes and frequencies are coded together. The variables involved in the problem are mapped in a coded form. The coding scheme is the key issue, as it links the real world problem to the GA problem. The GA manipulates the coded representation of the problem. The most common coding method is to transform the variables to a binary string. GA performs the best when binary coding is adopted (Goldberg, 1989). The length of the string is determined as per the desired solution accuracy. In this study, routes and coordinating frequencies of each pair are coded into a single string. Figure 3 indicates typical binary digits coding for route no. 5 and route no. 3 with frequencies 6 and 21 per hour. The first four bits show the route and last six bits show the corresponding frequency in a string. Once the coding of variables has been done, the corresponding point can be found using a fixed mapping rule. Usually the following linear mapping rule is used (Goldberg, 1989).

\[ X_i = X_i \text{. min} + \frac{X_i \text{. max} - X_i \text{. min}}{2^l_i - 1} \times \text{Decoded Value (S_i)} \]

Where,

\[ X_i \text{. min} = \text{is lower bound on decision variable } X_i \]
\[ X_i \text{. max} = \text{is upper bound on decision variable } X_i \]

The variable \( X_i \) is coded in a substring \( S_i \) of length \( l_i \). The decoded value \( (S_i) \) is \( \sum_{j=0}^{l_i} 2^j S_i \), where \( S_i \in (0,1) \) and the string is represented as \( (S_{i.1}, S_{i.2}, \ldots\ldots\ldots S_{i.2}, S_{i.3}, S_{i.0}) \). When all the decision variables are decoded using the above mapping rule, the function value can also be calculated by substituting the variables in the given objective function \( f(x) \). The obtainable accuracy of the variable for a \( l_i \)-bit coding is

\[ \frac{X_i \text{. max} - X_i \text{. min}}{2^{l_i} - 1} \]
Having decided on the representation, the first step in the simple GA is to create an initial population or initial parent pool of solutions. This is usually done by generating the number of individuals using a random number generator. It is advantageous to have as large a parent-pool size as possible to increase the number of schemata being processed per iteration. However there are obvious constraints of computer memory space and processing time. An optimal parent-pool size has to be determined for a given problem (Deb, 1995). Once the parent pool size is determined, the parent pool of solutions is generated by a random process.

The GA parameters and associated processes were tuned for the proposed objective function. Different values of crossover probabilities ranging from 0.1 to 1.0 were tested with mutation probabilities varying from 0.01 to 0.15. The best combination of crossover and mutation probabilities were selected based on value of penalized objective function (lowest), load factor (with in prescribed limits) and unsatisfied demand (as minimum as possible). The pool size is also decided on the basis of number of generations required for convergence. Least number of generations satisfying above criteria is selected. Thus tuning of parameters is done by several trials with different sets of values and following values were adopted.

Size of each String: 10 (6 bits for frequencies and 4 for routes)

(For schedules: Say max frequency = 60 (1 bus per minute), Min. frequency = 1 (1 bus per hour)
Obtainable accuracy = 1 gives value of $l_i = 5.9$ i.e. ‘6’ bits
Similarly for routes: Say (for 15 nodes) if maximum number of routes = 15, min route = 1
Thus for obtainable accuracy of $= 1, l_i = 3.9$ i.e ‘4’)

Pool size: 60
Type of crossover: Uniform
Crossover Probability: 0.95
Type of Mutation: Random
Mutation Probability: 0.10
Type of selection: Roulette
Random Seed: 1
Number of Generations: Till Convergence
4. Results and Discussions

Well scattered destinations covering the entire influence area of the DART station were selected as potential destinations and k-paths were developed (k = 5 for the case study) from DART station to these potential destinations as per codes given in Table 1. The penalised objective function (addition of objective function and penalties) and developed k-paths are used with Genetic Algorithms. Table 2 shows the percentage unsatisfied demand with different groups of destinations. It can be seen in the table that a higher percentage of demand is satisfied when nodes are dispersed and away from the DART station for the typical network under consideration. The 100% demand is satisfied when potential destinations are Cabinteely (11), Lough Linstown (12), Mount Merrion (13), Dundrum (14) and Sandyford (15). Thus these nodes are selected as potential destinations and feeder routes are developed for them from the DART station. Table 3 gives developed ‘k’ paths with their lengths in terms of travel time in ‘minutes’ for the above potential destinations.

Figure 4 shows the developed feeder route structure of Dun Laoghaire after the application of GA. It can be seen that there are five routes starting from Dun Laoghaire DART station (1), as follows:

Route no. 1: \[1 \to 3 \to 11\] (length 4.5 Km.)
Route no. 2: \[1 \to 2 \to 3 \to 16 \to 12\] (length 6.75 Km.)
Route no. 3: \[1 \to 4 \to 6 \to 7 \to 13\] (length 8.00 Km.)
Route no. 4: \[1 \to 6 \to 7 \to 8 \to 14\] (length 10.25 Km.)
Route no. 5: \[1 \to 2 \to 5 \to 10 \to 9 \to 15\] (length 9.00 Km.)

The lengths of developed feeder routes are within the adopted range of the minimum (2.5 km) and maximum (15 km). It can be seen that route numbers 3 and 4 overlap partially and that node 3 is common to both routes 1 and 2. The overlapping aspect of the routes has been analysed successfully.

Table 4 indicates coordinated schedules of feeder buses for the existing schedules of DART and the average load factors on each route. It can be seen in the table that the load factor varies in the range of 0.27 to 0.83. These load factors will further improve due to local demands at en-route stops. The local demand is not considered because the routes are designed for feeder buses from the DART station. Hence the satisfaction of the demand generated by the DART station is of prime concern. The overall load factor attains a value of 0.57 which is a much improved value against the present scenario in which the load...
factor remains in the range of 0.2 to 0.3 even during peak hours. In the case study, the capacity of feeder buses is taken as 74 (Scott Wilson, 2000) which is higher than the number of DART commuters transferring to buses. The load factor will be more (greater than 1) if coordinating buses are held for longer time. Commuters from many DARTs will be able to seek transfer to a particular bus due to longer holding periods. Longer holding increases the transfer time between DARTs and feeder buses which is not desirable to users and will also increase the value of the objective function. Thus the two contradictory conditions regarding higher load factors for operator and lower transfer time for users are satisfied by striking a balance between the two. As the load factor decreases below 1, higher values of penalties are imposed. Thus the load factors obtained in the study are less than 1 but transfer time is well within the desirable limits which is evident in Table 5. Buses with lower seating capacity will improve the load factor. Also the load factor will be higher if the model is implemented in places where higher numbers of train commuters seek transfer to buses.

Shrivastava and O’Mahony (2005) have developed feeder routes and coordinated schedules for the same study area by decomposing the problem in two stages: one for development of feeder routes and another for schedule coordination. The load factors for different routes were in the range of 0.16 to 0.45 with an overall load factor of 0.36. Thus the load factors and overall load factor by the proposed model have been improved compared with those in the earlier model (Shrivastava and O’Mahony, 2005). Total number of buses required will be 24 with 3, 6, 5, 5 and 5 buses on routes 1, 2, 3, 4 and 5 respectively for the peak period. However lesser buses will be required during off-peak hours.

Table 5 shows route wise details of the waiting time and by considering all the routes together. If all the routes were viewed together then it can be seen that 42% of the demand is satisfied within 5 minutes of effective waiting and 29% is satisfied between 6 to 10 minutes of waiting. Thus more than 70% of demand is satisfied within 10 minutes of effective waiting time and the entire demand is satisfied within 20 minutes of effective waiting. In the present scenario, the majority of commuters have to wait 20 minutes or even more for buses and the load factor is also less than 0.3 most of the time even during the peak hours. Looking at table 5 for route details, it can be seen that on route number 2, 62% of demand is satisfied within 5 minutes of effective waiting and the entire demand is satisfied with in 10 minutes of effective waiting. Route number 2 carries about 31% of the total trips thus making this route a maximum
trip carrier. For all the routes, more than 50% of demand is satisfied within 10 minutes of effective waiting time.

In the developed feeder route structure, the lengths of routes vary between 4.5 km to 10.25 km and 100% demand is satisfied without any transfer. If a similar exercise was carried out by identifying the influence area of all stations shorter feeder routes would be developed which would be better for maintaining schedules (Shrivastava and Dhingra, 2001). This is due to the fact that a node may be connected to more than one DART station and its connectivity will certainly be better with a shorter connecting length from one particular station only. It can be seen in Figure 4 that destinations like Stillorgan (8), Mount Merrion (13) and Dundrum (14) are closer to Blackrock DART station as compared to Dun Laoghaire. Thus feeder routes for these destinations from Blackrock will be shorter.

In this research data collection is done during morning peak period for one day only. This has been done in order to develop the methodology and to demonstrate the results. However to develop actual feeder route network the influence area of each DART station should be identified and data collection should be carried out on large scale. The traffic surveys should be conducted during peak and off peak periods of day for at least a week at all the DART stations to collect more reliable and accurate data. The frequency (coordinated schedules) of buses will vary as per demand in different periods.

5. Conclusions

In this study, feeder routes and frequencies of coordinating buses leading to coordinated schedules with the existing schedules of DARTs are optimised using Genetic Algorithms. Thus instead of decomposing the problem in two stages of development of feeder routes and coordinated schedules the two are optimised together and are complementary to each other. Decomposition of the problem in two stages had been the common practice for many network design problems in past. The adopted approach in this research gave better results in terms of improved load factors as compared to an earlier approach adopted by the authors (Shrivastava and O’Mahony, 2005) for the same study area and selected destinations. On the policy side, it can be decided to adopt a fully integrated system with longer journeys by DARTs and buses can feed the local areas with feeder route network and coordinated schedules as suggested in the model. The following conclusions can be drawn from the modelling exercise attempted in this study.
1. The selection of nodes as potential destinations plays a very important role for successful development of feeder routes for the type of network under consideration. The network which is considered for analysis has nodes well scattered in the influence area and destinations having higher demands are closer to origin (DART station). Some of the destinations away from DART station have limited connectivity with other nodes. Selection of potential destinations away from the origin and well dispersed in the influence area develops feeder route structure which satisfy the entire demand without any transfer.

2. The Genetic Algorithms are found to be very effective in optimization of feeder routes and coordinated schedules simultaneously. Routes and schedules (frequencies) can be coded into single strings and GA parameters including population size can be tuned for the objective function. These concatenated strings are found to be very efficient in simultaneous development of feeder routes and frequencies leading to coordinated schedules.

3. As a policy decision, the influence area of each station can be identified and the modelling exercise can be repeated, thus a fully integrated system can be developed in which DARTs can work as a main line haul facility and buses can feed the local areas. The proposed methodology is able to design feeder routes without any further transfers and the entire demand is satisfied directly from the DART station. The model also takes into account the overlapping aspect of different routes successfully.

Another policy decision can be adopted for schedules of feeder buses. The synchronised services of buses as developed by model can be adopted. The developed coordinated schedules and feeder routes have following advantages.

a. The user cost (transfer time between DARTs & buses + in-vehicle time) and operator’s cost (vehicle operating cost) is minimised. The model is also able to maintain a better level of service (constraint for max load factor), economical operation (minimum level of service), able to restrict the fleet size (fleet size constraint) and increases the probability of availability of buses to the majority of commuters (unsatisfied demand constraint). Thus the model incorporates real life objectives and constraints.

b. On average, the coordinated schedules and feeder routes are able to satisfy more than 50% of demand within ten minutes waiting and the entire demand is satisfied within twenty
minutes of waiting with an overall load factor more than 50%. On some routes, 80 to 90 percent of demand is satisfied within ten minutes of waiting and many routes have load factors even more than 0.7. In the present scenario (not optimised), the load factor hardly attains a value more than 0.3 and also the average waiting time is in the range of 20 minutes or more. Thus the proposed model judiciously balances waiting time and load factors for feeder buses.

4. The load factors in this case study are less than 1 for most of the feeder routes. One of the reasons for these low values of load factors is the higher capacity of buses as compared to the demand. Thus one of the policy decisions can be to allow feeder buses with lower seating capacity. Operation of minibuses would be quite effective and economical in such situations.

5. The proposed model optimises feeder routes along with coordinated schedules in contrast to earlier modelling exercises in which feeder routes may remain at suboptimal level. In addition to this the proposed model is able to provide higher load factors on various routes and also the overall load factor is improved considerably as discussed above. Thus the proposed model is better than the authors’ earlier model for the same study area in which problem is decomposed in two separate stages of routing and scheduling (Shrivastava and O’Mahony, 2005).

It has been indicated in the literature review that studies towards modelling integrated public transport system are very limited. Thus it can be claimed that the proposed modelling exercise is a specific contribution towards realistic modelling of integrated public transport system which involves most of the practical realities and constraints.

References


19. Maps and Zones at Home page of Irish rail for planning the journey by DART ,


Table 1  Potential Demand to Various Destinations

<table>
<thead>
<tr>
<th>Node No. (code)</th>
<th>Destinations</th>
<th>Potential demand to various destinations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>7 – 8 a.m.</td>
</tr>
<tr>
<td>1</td>
<td>Dun Laoghaire DART Station</td>
<td>00</td>
</tr>
<tr>
<td>2</td>
<td>Dun Laoghaire College</td>
<td>39</td>
</tr>
<tr>
<td>3</td>
<td>Sallynoggin</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>Monks town</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Deans Grange</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>Temple Hill</td>
<td>02</td>
</tr>
<tr>
<td>7</td>
<td>Black Rock</td>
<td>08</td>
</tr>
<tr>
<td>8</td>
<td>Stillorgan</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>Leopards town</td>
<td>02</td>
</tr>
<tr>
<td>10</td>
<td>Foxrock</td>
<td>02</td>
</tr>
<tr>
<td>11</td>
<td>Maple Manor / Cabinteely</td>
<td>02</td>
</tr>
<tr>
<td>12</td>
<td>Lough Linstown</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>Mount Merrion</td>
<td>02</td>
</tr>
<tr>
<td>14</td>
<td>Dundrum</td>
<td>06</td>
</tr>
<tr>
<td>15</td>
<td>Sandyford</td>
<td>03</td>
</tr>
<tr>
<td>16</td>
<td>Rouches Town Avenue</td>
<td>02</td>
</tr>
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</table>
Table 2  Selection of Potential Destinations

<table>
<thead>
<tr>
<th>Potential Origin</th>
<th>Potential Destinations</th>
<th>Unsatisfied demand %</th>
<th>Direct demand satisfied, %</th>
<th>Total demand satisfied %</th>
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<td>73.02</td>
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<td>26.47</td>
<td>73.53</td>
<td>73.53</td>
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<td>09.89</td>
<td>90.11</td>
<td>90.11</td>
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<td>96.02</td>
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<td>00.00</td>
<td>100</td>
<td>100</td>
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Table 3  Developed ‘K’ paths between DART station and Potential Destinations

<table>
<thead>
<tr>
<th>Origin</th>
<th>Potential Destination</th>
<th>Nodes in ‘k’ paths (k=5)</th>
<th>Travel time min</th>
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<tr>
<td>1</td>
<td>11</td>
<td>1, 3, 11</td>
<td>18</td>
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<td></td>
<td>1,4,6,7,13</td>
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<td>14</td>
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<td>1,7,8,14</td>
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Table 4  Details of Bus Schedules with Load Factors

<table>
<thead>
<tr>
<th>DART Timings</th>
<th>Bus Timings</th>
<th>Av. Load Factors and number of buses required</th>
<th>Over all load factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Bound DARTS</td>
<td>South Bound DARTS</td>
<td>Route 1</td>
<td>Route 2</td>
</tr>
<tr>
<td>08.08</td>
<td>08.02</td>
<td>8.07</td>
<td>8.07</td>
</tr>
<tr>
<td>08.15</td>
<td>08.09</td>
<td>8.27</td>
<td>8.19</td>
</tr>
<tr>
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<td>8.47</td>
<td>8.31</td>
</tr>
<tr>
<td>08.29</td>
<td>08.25</td>
<td>8.43</td>
<td>8.52</td>
</tr>
<tr>
<td>08.33</td>
<td>08.31</td>
<td>8.55</td>
<td>-</td>
</tr>
<tr>
<td>08.38</td>
<td>08.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>08.43</td>
<td>08.45</td>
<td></td>
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</tr>
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<td>08.49</td>
<td>08.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>08.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trains after 9 a.m.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Buses to be scheduled after 9 a.m.

Total number of buses needed = 24
### Table 5  Waiting Time Details of Passengers

<table>
<thead>
<tr>
<th>Duration of Delay in Minutes</th>
<th>Route No : 1</th>
<th>Route No : 2</th>
<th>Route No : 3</th>
<th>Route No : 4</th>
<th>Route No : 5</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 5</td>
<td>31</td>
<td>62</td>
<td>41</td>
<td>31.5</td>
<td>32.5</td>
<td>42</td>
</tr>
<tr>
<td>6 - 10</td>
<td>24</td>
<td>38</td>
<td>43</td>
<td>22</td>
<td>20</td>
<td>29</td>
</tr>
<tr>
<td>11 - 15</td>
<td>27.5</td>
<td>nil</td>
<td>16</td>
<td>28</td>
<td>29</td>
<td>18.5</td>
</tr>
<tr>
<td>15 - 20</td>
<td>17.5</td>
<td>nil</td>
<td>nil</td>
<td>18.5</td>
<td>18.5</td>
<td>10.5</td>
</tr>
<tr>
<td>&gt; 20</td>
<td>nil</td>
<td>nil</td>
<td>nil</td>
<td>nil</td>
<td>nil</td>
<td>nil</td>
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</tbody>
</table>
Figure 1 Existing DART line along stations (Map and Zones at Home Page of Irish Rail)
Optimization of penalized objective function using Genetic Algorithms

Objective function: Minimization (Transfer time between DARTs and Buses + in Vehicle time + Vehicle operating cost)

Constraints: Related to Minimum and Maximum load factors, fleet size and unsatisfied demand

TRAFFIC SURVEYS

Surveys for assessing existing distribution of DART Commuters on Different Modes

Assessment of potential demand to different destinations from DART station using willingness to shift surveys

Existing Road Network

Potential O - D Matrix

Selection of Potential destinations as per demands and locations in the influence area

Development of K – paths between DART station and potential destinations

Optimization of penalized objective function using Genetic Algorithms

Existing DART timings

Optimized Feeder routes and coordinated schedules

Is entire demand Satisfied?

Yes

Are the lengths of feeder routes with in acceptable limits?

Yes

No

Optimized feeder routes and coordinated schedules to be adopted

No

Figure 2 Proposed methodologies for development of routes and schedules together
<table>
<thead>
<tr>
<th>Pair no 1</th>
<th>Pair no 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route</td>
<td>Frequency</td>
</tr>
<tr>
<td>0 1 0 1</td>
<td>0 0 0 1 1 0</td>
</tr>
</tbody>
</table>

Route no 5 | Frequency 6
Route no 3 | Frequency 21

Figure 3  Binary digit coding
FIGURE 4   Developed feeder route network for Dun Laoghaire DART station