Time-Series Modeling For Forecasting Vehicular Traffic Flow in Dublin

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ABSTRACT

The traffic flow at an arterial intersection in a congested urban transportation network in the city of Dublin is modelled in this paper. Three different time-series models, viz. random walk model, Holt-Winters’ exponential smoothing technique and seasonal ARIMA model are used for modeling of traffic flow in Dublin. Simulation and short-term forecasting of univariate traffic flow data are done using these models. The data used for modeling are obtained from loop-detectors at a certain junction in the city center of Dublin. Seasonal ARIMA and Holt-Winters’ exponential smoothing technique give highly competitive forecasts and match considerably well with the observed traffic flow data during rush hours.
INTRODUCTION

With the advent of Intelligent Transportation System (ITS), short-term forecasting and simulation of traffic flow are becoming important. Optimisation of the operational parameters related to traffic management in a transport network can be achieved by short-term forecasting. In most of the major cities and highways large amount of traffic data are available from the network of sensors and devices located at different sites in a transport network. Using the fixed-location traffic detector systems, traffic flow (volume/hour), speed and lane occupancy and other traffic stream characteristics are usually recorded and are available for analysis. Analysis of these data for short-term forecasting of traffic characteristics using different non-conventional techniques such as, time delay neural network (TDNN) using genetic algorithm (1), approximate nearest neighbour (ANN) nonparametric regression (2), and, time-series modeling has been of research interest. In time series modeling several techniques like the ‘Box & Jenkins techniques’ (3), multivariate analysis (4), application of subset ARIMA model (5), application of seasonal ARIMA model (6), exponential smoothing model (7) etc. are applied by several researchers.

Most of the above-mentioned modeling techniques are applied to the traffic flow data collected from the expressways, interstate and intrastate highways, which are essentially urban freeways. In this paper an effort has been made to find out the performance of these conventional and non-conventional techniques in simulation and short-term forecasting of traffic flow while applied to the congested and comparatively narrower roads inside a city. Generally in a busy city, weekday traffic flow patterns are very much unlike the weekend traffic flow patterns. This is also valid in Dublin, a very busy and cosmopolitan capital city. While modeling the traffic flow, this particular factor has been taken into account in this paper. Therefore, unlike some other literatures in this field (6), only a daily periodicity is considered in this paper. A few plausible models are applied and a comparison of the models has been done suggesting the relative suitability of different techniques in context of the simulation and short-term forecasting of traffic flow in the streets of a rapidly growing city like Dublin.

In this paper three different conventional and non-conventional time-series techniques are used for modeling the traffic flow data in Dublin City. The random walk model, Holt Winters’ exponential smoothing model and the seasonal ARIMA model are used for the purpose of modeling and are compared in this context. The traffic flow data are obtained from the loop-detectors at many junctions and road crossings at the city-center of Dublin. From this network of loop-detectors, traffic volume for each fifteen-minute interval is recorded at a number of sites (mostly junctions). The traffic flow data, to be modeled, are taken from a single site in the city. As the city-center junctions have very similar weekday traffic flow patterns only a single site is considered. Since a univariate short-term prediction problem to be considered, the spatial variability of the data is not taken into account. Suitability of the models in short-term forecasting is judged by comparing the error estimates like root mean square error (RMSE) or maximum absolute percentage error (MAPE) of the different models.

2.0 TRAFFIC FLOW AT A JUNCTION

The data to be modeled in this paper are obtained from the loop detectors at a particular junction of the city-center of Dublin. The junction is TCS 183. A map of the junction is given in figure 1. The junction is a four-legged junction, with one-way traffic on two approaches. Tara Street (one-way) has four lanes, with traffic flowing from south to north. The traffic volume passing through Tara Street in each 15-minute interval, measured from the loop-detectors number 1, 2, 3, 4 are continuously recorded. The data obtained collectively from these four detectors, are used for the modeling. As the weekend travel behaviour is very much unlike the travel behaviour in the weekdays, the modeling is essentially carried out on the data observed during weekdays. Since, the data set used does not contain any missing data, no special treatment for missing data is required to be utilised here.

The data used for modeling were recorded from 3rd November 2003 midnight to 30th November 2003 midnight, excluding the weekends. 96 observations are obtained in each day. The total number of observations is 1920, i.e., in total, 20 days of data are used. The plot of data is shown in figure 2. Fifteen minutes are considered as one unit in time index in this plot.

The data show definite seasonality in pattern over a period of 24 hours. This leads to the idea of fitting a seasonal time-series model. Different types of time-series models are used here for modeling the data, for simulation and short-term forecasting.
3.0 ANALYSIS OF THE DATA

For the purpose of time-series modeling, the data are analysed for getting an overview about the characteristics of the series. The software MINITAB 13 is used to calculate the statistical functions required in the paper.

To convert the traffic flow data series to a zero-mean sequence, the mean is subtracted from each observation. The zero-mean process or the centered traffic data set is to be processed further to make it ‘stationary’ in nature. In this paper ‘stationarity’ means ‘second-order stationarity’. According to the ‘second-order stationarity’ condition, the mean $\mu(t)$ should be equal to a constant, $\mu$, for all $t$ and auto-covariance $\gamma(t, s)$ depends only on $|t-s|$, the absolute difference between $t$ and $s$ ($s$ is the lag at which auto-covariance is calculated) (8).

A correlogram of the centered data is plotted in figure 3. The two dotted lines, nearly parallel to the x-axis of the plot, denote the 95% confidence interval for the ACF (autocorrelation function) (8). If the value of the ACF is within these lines, then it can be considered to be negligible or equal to zero. These kinds of limiting lines are provided in each of the plots for autocorrelation and partial autocorrelation functions (8). The correlogram shows that the ACF has considerably large values repetitively, after a particular interval of time. The ACF is very much like a sine wave with a time period of 96 observations. This is a clear indication of the seasonality of the data. The PACF (partial autocorrelation function) of the centred traffic data (figure 3) is plotted as well. This plot confirms the seasonality i.e. the repetitive correlation at lag 96 (i.e. comparison of the 1st observation with the 97th observation).

From the correlogram it is quite evident that the zero-mean process is also non-stationary in nature. As the series is seasonal in nature, a seasonal difference is taken to remove this non-stationarity. The correlogram and ‘partial autocorrelation function’ plots of the differenced (figure 4) and centered traffic data show that the ACF or PACF values drop to zero (i.e. within the limiting lines) rapidly. Hence, the series is now stationary in nature.

4.0 MODELS USED

The traffic flow data have been modeled using three approaches:
1. Random walk approach;
2. Holt-Winters’ trend and seasonal smoothing technique;
3. Seasonal ARIMA model;

Out of these three approaches, the original data series is used only in Holt-Winters’ trend and seasonal smoothing technique. The differenced and centered traffic flow data series is used for the other two techniques. All three models are used for forecasting and the forecasts and the relative errors from each model are calculated to find the most suitable modeling technique for this particular traffic data.

4.1 Random Walk Model

The random walk model is one of the most widely used naïve method of time-series modeling. The random walk approach uses the most recent actual observation for forecasting the next value. This approach is quite popular in economic forecasting.

The random walk model does not take into account the effect of historical observations. Though random walk model can be used directly on the non-stationary data, here the centered and the differenced data are used to take into account of the seasonal nature of the series.

To include seasonality the following model is used.

\[ y_t^* = y_t - y_{t-96} \]  (1)

\[ y_t^* = y_{t-1}^* + \epsilon_t \]  (2)

where, $\epsilon_t \sim N(0, \sigma^2)$

Hence, \[ y_t = y_{t-1}^* + y_{t-96} - y_{t-97} + \epsilon_t \]  (3)

Here $y_t$ is the observed data at an instant of time $t$; $\epsilon_t$ is the random error at time instant $t$, generated from the normal distribution, with zero mean and variance $\sigma^2$. 
This model is good for one-step-ahead prediction. As the seasonal difference of the data is used for generating the forecasts, the forecasts for the next day (i.e. 96 points of future predictions) are essentially one-step ahead predictions.

The point predictions along with 95% prediction interval are given in figure 5. Traffic volumes per fifteen minute (with the time indexed, at an interval of 15 minute) are plotted here for the original and forecasted cases. The RMSE and MAPE (9) errors are given and compared with other models in table 1 and in section 5.0.

4.2 Holt-Winters’ Trend and Seasonal Smoothing

‘Holt-Winters’ trend and seasonal smoothing technique’ is a generalized version of exponential smoothing for dealing with variations in ‘trend’ and ‘seasonality’ both over time. The technique of exponential smoothing is essentially a forecasting method but the method is not based on a probability model, and hence can be considered as ‘ad-hoc’ (8). ‘Exponential smoothing’ (ES) is basically the ‘weighted moving average’ technique, where the weights are decreasing exponentially as the observations are getting older. So, the maximum weight is given on the most current observation.

In ‘Holt-Winters’ trend and seasonal smoothing technique’, the weights are reduced from two directions, viz. seasonally and historically. This particular technique of ES is particularly suitable for data with seasonality and trend both. In this method, three equations are used to account for the ‘level’, ‘trend’ and ‘seasonality’. If $y_t$ is the observed data at the time instant $t$, $\alpha$, $\beta$, $\gamma$ are the level, trend and seasonal smoothing parameters respectively and $s$ is the number of periods in a seasonal cycle, then the equations for the ‘additive’ model are:

- For updating the level index $L_t$,
  $$L_t = \alpha \frac{y_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + b_{t-1})$$

- For updating the trend index $b_t$,
  $$b_t = \beta(L_t - L_{t-1}) + (1-\alpha)(L_{t-1} + b_{t-1})$$

- For updating the seasonal index $S_t$,
  $$S_t = \gamma \frac{y_t}{L_t} + (1-\gamma)S_{t-s}$$

Using these three indices, the equation for forecast, $F_{t+m}$ for the additive model is,

$$F_{t+m} = L_t + b_t m + S_{t-s+m}$$

where, $m$ is number of periods ahead.

This model is now applied to the available traffic flow data. The traffic flow data show definite patterns of seasonality, but no considerable trend. As the above-mentioned variation of ES is particularly good in modeling data with seasonality, hence this technique is also suitable for modeling the traffic flow data. The traffic flow data over a month follow similar values and patterns for each day. So, the seasonality considered for this model is the ‘additive’ seasonality.

The values of the smoothing parameters are taken as, $\alpha = 0.05$, $\beta = 0.02$, $\delta = 0.03$ for modeling the original traffic flow data. The optimum values of the smoothing parameters are found out by minimizing the MAPE error. The forecasts from the model are discussed in section 5.0.

4.3 Seasonal ARIMA Process

4.3.1 Simple Autoregressive Moving Average (ARIMA) Process (9)

A simple ARIMA model constitutes of three parts, ‘AR’, i.e. autoregressive part; ‘I’, i.e. differencing part; ‘MA’, i.e. moving average part;

For simplicity, the second part is discussed first. ‘Differencing’ is essentially a tool to eliminate trend in a time series data. It can be described as one of the filter/ transformation techniques for removing ‘non-stationarity’ from any time-series data.
The ‘first difference’ $y'_t$ of any time-series data is,

$$y'_t = y_t - y_{t-1} \quad (8)$$

In an ‘auto regressive’ process, each time-series observation ‘$y_t$’ is defined in terms of its predecessors, ‘$y_s$’, for $s < t$, by the equation,

$$y_t = \sum_{l=1}^{p} \alpha_l y_{t-l} + Z_t \quad (9)$$

where, $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_p$ are the coefficients of the auto regressive process of the order $p$.

A ‘moving average’ process is simply a finite linear filter applied to a white noise sequence $\{Z_t\}$, of the form

$$y_t = Z_t + \sum_{j=1}^{q} \beta_j Z_{t-j} \quad (10)$$

where, $\beta_1, \beta_2, \beta_3, \ldots, \beta_q$ are the coefficients of the moving average process of the order $q$.

Combining these three components and using ‘backshift operator’, $B$, the equation representing an ARIMA ($p, d, q$) model is,

$$\phi(B)(1-B)^d y_t = \theta(B) Z_t \quad (11)$$

where, $Z_t$ is a white noise sequence;

$\phi$ is a polynomial of degree $p$, i.e. $\phi(B) = (1 - \alpha_1 B - \alpha_2 B^2 - \alpha_3 B^3 - \ldots - \alpha_p B^p)$ and $\theta$ is a polynomial of degree $q$, i.e. $\theta(B) = (1 - \beta_1 B - \beta_2 B^2 - \beta_3 B^3 - \ldots - \beta_q B^q)$.

In ARIMA ($p,d,q$), $p$ denotes the order of the AR process, $d$ denotes the order of differencing and $q$ denotes the order of MA process.

### 4.3.2 Seasonality and ARIMA Process (10)

If the time-series data to be fitted in an ARIMA model has some intrinsic seasonality, then instead of simple ARIMA, a seasonal ARIMA model can be used. There are two types of seasonal models, additive and multiplicative. Here the multiplicative seasonal ARIMA model ($p, d, q$)$P, D, Q_s$ is used. In the multiplicative model the non-seasonal part ($p, d, q$) and the seasonal part ($P, D, Q$) are multiplied together. The equation used for the multiplicative seasonal ARIMA model is as follows:

$$\phi(B)(1-B)^d (1-B^S)^d y_t = \theta(B) \Theta(B^S) Z_t \quad (13)$$

where, $\phi, \theta, \Theta$ have the same significance as described in the earlier section and $\Phi, \Theta$ are their seasonal counterparts, $S$ denotes the seasonality.

The centered traffic data set is used for ARIMA modeling using Box and Jenkins (11) methodology.

**Identification** The traffic data (time series) plot (figure 2) shows that the data set is seasonal in nature. Seasonal ARIMA process seems suitable for modeling and a seasonal difference of the data is taken to account for the non-stationarity of the data, as described in section 2.0. The correlogram of the differenced data shows no significant non-stationarity (see figure 4). Again from the ACF of the differenced data an exponential decay at lag 1 is very distinct. The correlogram suggests that the process must be seasonal at lag 96 and AR (1) or AR (2) must be a non-seasonal component of the model. Apart from that, MA (1) also seems to be a plausible seasonal component. Therefore, some variations of the ARIMA model of the nature of ($p, 0, q$)$P, 1, Q_{96}$ will give a suitable model for representing the traffic flow data.

**Estimation** In table 2, some of the different possible variations of the ARIMA models and the error estimates for each of the models are shown. The error estimates used here are MSE (mean square error) and AIC (Akaike’s Information Criterion). AIC is one of the most common penalized likelihood procedures (12).

$$\text{AIC} = -2 \log L + 2m \quad (14)$$

where, $m = (p+q+P+Q)$ and $L$ denotes likelihood. For difficulties often encountered in calculating the likelihood a useful approximation is used.

$$-2 \log L = n(1+\log(2\pi)) + n \log \sigma^2 \quad (15)$$

where, $n$ is number of observations and $\sigma^2$ is the variance of the residuals.
For all the models discussed here, the number of observations is same. Hence, the first part of the likelihood, $n(1+\log(2\pi))$ is the same for all of them. So, in this paper while calculating AIC, the first part of the likelihood is eliminated. Due to this in the AIC column of the table 2, the value given is not exactly equal to actual value of AIC of the model. The model with minimum AIC value is the most suitable one. AIC is not a relative error and does not have much significance when used for comparing models from different contexts. Only when the AIC values of different models from the same context are compared, the best fit model with minimum AIC is found out.

From these error estimates, ARIMA (2,0,1)(0,1,1)$_{96}$ seems to be the most suitable model. Both the AIC and MSE values are minimum in case of this model. However the other models are quite competitive as well.

**Diagnostic Checking** The ‘residuals’ are calculated after fitting the model. The ACF and the PACF of the ‘residuals’ are plotted (figure 6). The ACF and PACF plots show that the residuals are essentially white noise, as less than 5% of the ACF or PACF values fall distinctly outside the limiting lines. So, no further prediction is possible using these values.

For the ARIMA model, the outliers are also required to be checked. Standardized ‘residuals’ (mean zero and variance one), are plotted to identify the outliers (see figure 7). The data (‘standardized residuals’), will be treated as outlier if it falls outside $\pm 3$. In the plot less than 2% of the data behave as outliers. So, the model can be considered as reasonably good fit.

The forecasts of the seasonal ARIMA model are discussed and compared with the forecasts from the other two models in the next section.

### 5.0 FORECAST

All three of the methods mentioned for time series modeling in section 4.0 are used to model the traffic flow data available from the four loop-detectors on the Tara Street, Junction no. TCS 183 forecasting 50 points in the future. The traffic flow data obtained on the 1st of December 2003, or data collected in next 12.5 hours (50x15 = 750 minute = 12.5 hours) are compared with these forecasts. As the random walk model is not comparable to the other two from the point of forecasting precision, hence this model is not used in the plot for comparing the forecasts. The ES and seasonal ARIMA forecasts along with the original observations are plotted in figure 8. Only the point forecasts are considered in each case and the prediction intervals are not employed. The point forecasts from the ES and seasonal ARIMA are considerably near to the original observations (see figure 8), where as the random walk method does not do so well (figure 5).

The seasonal ARIMA model is used on the centered traffic flow data. So, for comparing with the original non-centered observations the mean of the traffic flow data is added to all the forecasts from the model.

The errors from different models are compared in table 1. The RMSE and MAPE error show that the ES method is slightly better than the seasonal ARIMA model. The random walk model has larger error estimates. This inference is unlike the results when the same techniques are applied for modeling the traffic flow in urban freeways (7). The error estimates obtained, show quite high values of MAPE and RMSE. From figure 8 it is quite evident that there is a considerable difference at the first few points of the forecasts, from both the ES and seasonal ARIMA model, with the observed traffic flow data. If the data series is so adjusted that the forecasts are made from 6 am instead of 12 am, then the error estimates decrease considerably. The error estimates for 48 points (i.e. 12 hours) of forecasts, starting from 6 am on 1st of December, are given in table 3. The ES and seasonal ARIMA forecasts along with the original observations, for 6 am to 6 pm on 1st of December, are plotted in figure 9.

Another reason for the high values of RMSE and MAPE is sudden changes in the observed traffic flow data at some points of forecast. None of the models perform well in these regions. On eliminating those points the error estimates from both the ES and seasonal ARIMA model, decrease further (MAPE approximately equal to 5%).

### 6.0 CONCLUSION

The traffic flow data from a junction in Dublin city-center are modeled in this paper using three different time series techniques. The data show a bimodal nature, but the trough between the two peaks is not very distinct. The traffic volumes observed are considerably lesser in early morning hours than the values obtained in rush hours. Abrupt changes are also quite frequent due to abundance of incidents in a busy city street. The traffic flow counts over a month (except the weekends) show a definitive ‘daily’ seasonal nature. Based on this seasonality and trend (which is quite stable over a month but may change for greater time intervals) simulation and short-term forecasting
of the data is performed. From the comparison it is seen that the Holt Winters’ trend and seasonal smoothing technique is the best model for this junction. However, the seasonal ARIMA model is also quite reasonable. The error estimates obtained using all these models are considerably large compared to the results when the same models are applied on a freeway (7). This large error estimate is essentially due to the difference of the forecasts from the models with the actual data, in the first few points (i.e. early morning observations). The models perform very well while dealing with rush hour data (table 3 and figure 9). Again in cases of some abrupt changes in the observed traffic flow data, the models fail to match very well.

Here the models are applied only to a single junction in the city. Same techniques can be applied to model the other junctions of the city. Further research is being carried out on spatio-temporal simulation and forecasting of traffic flow in an urban transport network system.

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<thead>
<tr>
<th>MODEL</th>
<th>RMSE</th>
<th>MAPE</th>
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<tbody>
<tr>
<td>Exponential Smoothing</td>
<td>37.8</td>
<td>22.11%</td>
</tr>
<tr>
<td>Seasonal ARIMA (2,0,1)(0,1,1)_y</td>
<td>41.89</td>
<td>28.55%</td>
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<td>Random Walk Model</td>
<td>85.82</td>
<td>85.2%</td>
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</table>
TABLE 2  Error Estimates from Different ARIMA Models

<table>
<thead>
<tr>
<th>ARIMA MODEL</th>
<th>COEFFICIENTS</th>
<th>MSE</th>
<th>AIC</th>
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<tbody>
<tr>
<td>(1,0,1)(1,1,1)_{96}</td>
<td>AR1 0.7500  AR2 0  MA1 0.3491  MA2 0  AR96 0.8348  MA96 -0.0928</td>
<td>1441</td>
<td>13965.6</td>
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<tr>
<td>(2,0,1)(0,1,1)_{96}</td>
<td>AR1 1.2048  AR2 -0.2649  MA1 0.7905  MA2 0  AR96 0  MA96 0.8513</td>
<td>1434</td>
<td>13963.495</td>
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<tr>
<td>(1,0,2)(0,1,1)_{96}</td>
<td>AR1 0.8667  AR2 0  MA1 0.4625  MA2 0.1186  AR96 0  MA96 0.8499</td>
<td>1440</td>
<td>13964.611</td>
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### TABLE 3  Comparison of Two Time Series Models in The Rush Hours

<table>
<thead>
<tr>
<th>MODEL</th>
<th>RMSE</th>
<th>MAPE</th>
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</thead>
<tbody>
<tr>
<td>Exponential Smoothing</td>
<td>61.47</td>
<td>11.22%</td>
</tr>
<tr>
<td>Seasonal ARIMA (2,0,1)(0,1,1)$_{se}$</td>
<td>61.93</td>
<td>11.28%</td>
</tr>
</tbody>
</table>
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