Recent advances in the fatigue design of steel bridges submitted to aleatory traffic loads

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**ABSTRACT:** This work presents recent developments in the fatigue design of bridges submitted to aleatory traffic loads. The purpose is to perform fatigue design according to Eurocode fatigue model 5 but replacing real-life measurements by Finite Element computations. Synthetic traffic loads are generated according to prescribed probability density functions representative of the real traffic. A Monte Carlo approach is employed to estimate the damage induced on the structure.

1. **INTRODUCTION**

   Nowadays, steel road bridges also need to fulfill fatigue requirements. For this kind of structures, the Eurocode EC (2003) provides simplified fatigue models that serve as basis for the design. However, the use of these models often results in too conservative designs. Alternatively, the Eurocode accepts on-site measurements to be employed as input for the computations (fatigue model 5). Of course, this approach is complex and expensive to set up, and simply impossible to pursue in the case of a new bridge. The aim of the approach presented here is to perform fatigue design according to Eurocode fatigue model 5 but replacing real-life measurements by Finite Element computations. Synthetic traffic loads are generated according to prescribed probability density functions representative of the real traffic. A Montecarlo approach is then employed to estimate the damage induced on the structure, based on Eurocode SN curves EC (2005). This work takes inspiration from the work of Baptista (2016).

2. **ALEATORY TRAFFIC GENERATION**

2.1. **Traffic parameters**

   In the following, we adopt the same hypotheses as Baptista (2016), but the framework that we developed is totally general and can consider different hypotheses (in terms of probability density functions, flux, number of lorries, etc...).

2.1.1. **Simulated duration**

   The simulated traffic duration is one day since statistical data are often provided on a daily basis. Simulating multiple days of traffic goes back to running multiple one-day simulations.

2.1.2. **Traffic type**

   During a working day, we must distinguish between rush hours (usually 6am-10am, 4pm-8pm) and off-peak hours. During off-peak hours,
traffic is considered as free flow, while during rush hours, traffic is considered as an alternating sequence of free-flow phases (10 minutes) and jammed phases (5 minutes).

According to Baptista (2016), free-flow traffic can be characterized through a Gamma distribution (Figure 1):

\[
f(x) = \begin{cases} 
\frac{1}{\Gamma(a)\beta^a} (x-a)^{a-1} e^{\frac{(x-a)}{\beta}}, & x \geq a \\
0, & x < a 
\end{cases}
\]

, where \(\Gamma(a)\) is the complete Gamma function and \(a, \alpha\) and \(\beta\) are user-defined shape parameters. On the other hand, jammed traffic can be modelled through a Beta distribution (Figure 2):

\[
f(x) = \begin{cases} 
\frac{1}{B(p,q)\beta^{p+q-1}} (x-a)^{p-1}(\beta + a - x)^{q-1}, & a \leq x < a + \beta \\
0, & x < a, x \geq a + \beta 
\end{cases}
\]

, where \(B(p,q)\) is the complete Beta function and \(p, q, a\) and \(\beta\) are user-defined shape parameters.

2.1.3. Number of vehicles

The total number of vehicles per day per lane is considered as a deterministic quantity. According to observations in Auxerre (see Baptista (2016)), we consider 32000 vehicles per day per lane, with a constant flux, \(Q\), of 1800 vehicles per hour. This leads to an observation window of \(32000/1800 \approx 18\) hours (typically between 4am and 10pm).

2.1.4. Number and type of trucks

As for the number of lorries, we must distinguish between slow and fast lanes. For slow lanes we consider 25% of the total traffic, i.e. 8000 trucks per day per lane, while for fast lanes we consider 10% of the total traffic, i.e. 3200 trucks per day per lane.

Concerning the type of trucks, we refer to the data recorded in Brothal (Baptista (2016)) (see Table 1). Each one of the five types is divided into two sub-types, to account for the empty vs loaded condition. For each type of truck, a bi-normal distribution (with known average and standard deviation) is employed to model the uncertainty on the weight. A relative frequency, considered as a constant deterministic quantity, is also given for each (sub-)type.

2.2. Inter-distances generation

The generation of the inter-distances between vehicles proceeds according to the following steps:

1. Choose the type of lane (slow vs fast). This dictates the proportion of trucks amongst the total amount of vehicles.
2. Perform a time loop over the day (typically between 4am and 10pm). The
day is subdivided into time steps of constant amplitude, $\Delta t$.

3. At each time step, choose the type of traffic to be modelled, free flow vs jammed (this depends on the time of the day).

4. Since the flux of vehicles is supposed constant, the number of vehicles can be computed as $N_{veh} = Q \times \Delta t$. In our example, let us suppose that $N_{veh} = 9$.

5. $N_{veh}$ aleatory values of inter-distances are generated from the probability density function corresponding to the current traffic type (Figure 3(a)). In this way an ensemble of $N_{veh}$ vehicles, with wanted inter-distances $d_i'$, has been generated. At this stage, it has not yet been decided which of these vehicles are cars and which are trucks.

6. Based on the type of lane one can decide how many of these vehicles are trucks, and thanks to the relative frequency given in Table 1, one can also determine the number of trucks of each (sub-)type. The position of the trucks in the ensemble of $N_{veh}$ vehicles is assigned aleatorily. All vehicles that are not trucks are cars (Figure 3(b)).

7. Cars can be eliminated, since their contribution to fatigue damage is neglected, and distances $d_i'$ between trucks can be computed as mere sums of the $d_i'$ between two successive trucks (Figure 3(c)).

8. Steps 4 to 7 are repeated for each time step until the end of the time loop.

An example of inter-distances generated during one day of traffic is depicted in Figure 4.

2.3. Lorries weights generation

Once the inter-distances generation is completed, the composition of the traffic is known. One can thus assign an aleatory weight to each truck, based on the normal distribution corresponding to its (sub-)type. In this way, a load train equivalent to the real traffic is obtained (Figure 3(d)).

2.4. Remarks

The procedure described above can sometimes lead to unrealistic situations.

One example is the case of compenetrating trucks if the distance between two trucks is lower than the leading truck length. To reduce the risk of this event, higher lower bounds for the Gamma
and Beta distributions could be employed, even though this has not been done in this work.

Another example is the generation of unrealistic weights, either too heavy or too light (the extreme case is a negative weight). To avoid this, the weight normal distributions are bounded to $\mu \pm 3\sigma$ ($\sim 99.8\%$ of trucks) in this work.

3. FATIGUE DAMAGE ESTIMATION

Once the aleatory traffic loads are generated, fatigue damage at a given point on the structure can be estimated as follows:

1. Compute the stress history at the point of interest.
2. Compute a histogram of stress variations, for instance by means of a rain-flow counting.
3. Compute the damage associated to each stress variation in the histogram using SN curves (EC (2005)).
4. Compute the total damage as the sum of the partial damages, according to Palgrim-Miner’s rule.

3.1. Influence lines approach

The computation of the stress history can be prohibitive from a computational viewpoint, due to the huge amount of time steps that would be necessary. Indeed, we found out that ~10 days of traffic should be simulated to achieve good convergence of the Montecarlo approach (see Section 3.2.1). Typical dimensions of trucks wheels are of the order of tens of centimeters, which, for a truck speed of 80 km/h, leads to a time step of ~0.004s. The simulation of ten days of traffic would then imply the computation of $\sim 10^8$ steps, which is unreasonable for a full-scale finite element model of a bridge.

Instead, we opted for an approach based on the use of influence lines. Only one finite element simulation is performed in which a unitary load moves along the whole bridge. Unitary influence lines for the target efforts/stresses can thus be obtained. The aleatory load train previously generated can then be applied on these influence lines to obtain the final response.

In practice, for a given verification point the effect of each truck passage is obtained by multiplying the unitary influence line by the weight of the truck and then shifting the signal in space by the corresponding distance of the truck from the origin of the load train. The full stress history at the verification point results from the superposition of all the signals (a linear interpolation is performed where needed. Refer to Figure 6 for an example). By doing so, the required large number of steps is computed only on a small amount of target stresses (instead of the full structure). Moreover, only cheap operations such as products and sums (instead of algebraic system solutions) are performed.

3.2. Example: sinusoidal influence lines

We now present the results obtained in the case of a sinusoidal unitary influence line, as depicted in Figure 5. This can ideally be thought of as the simplified model of a single-span bridge of length 100m. Of course, this represents no more than an academic example, but it is very useful to investigate the method proposed in this work.

![Figure 5: Sinusoidal unitary influence line.](image-url)
Figure 6: Loading and superposition of influence lines.

The full stress history of one day of traffic, resulting from the superposition of all the signals, is illustrated in Figure 7. The highest stresses are observed during rush hours. These are the moments when traffic jams occur, and hence when the number of trucks simultaneously present on the bridge is higher (cf. Figure 4).

Figure 7: Stress history of one day of simulated traffic.

The histogram of stress variations can be obtained by performing a rain-flow counting on the stress history. The one corresponding to the present example is reported in Figure 8. Such a histogram can then be employed to compute damage through SN curves, according to Palmgren-Miner’s rule.

Figure 8: Stress variation histogram (rainflow).

3.2.1. Convergence of the Montecarlo method
In the end, damage is the actual quantity of interest. Nonetheless, the use of a statistical approach, such as Montecarlo, demands to verify the statistical convergence of the analysis. In other words, one must verify if the number of realizations is sufficient to obtain trustful results. In this example, for the computation of damage we have arbitrarily chosen a fatigue class of 100.

Figure 9 shows the moving average of the daily damage by simulating from 1 to 250 days (~one year), for three different influence line lengths (or span lengths, in this example): 4m, 80m, and 200m. The values are normalized with respect to the average daily damage for 250 days (taken as reference).

The curves suggest that a minimum of about 10 days of traffic should be simulated to attain a satisfactory convergence (within 2%) in all cases, even though at least 150 days are needed to achieve a fully stabilized solution.

Faster convergence is observed for the shortest and longest influence lines. In fact, these represent two extreme cases:
- For very short influence lines (shorter than a truck), there is no interaction between trucks. The structure ‘feels’ only one truck at a time. This is the typical case of damage due to local effects. The distance between the trucks is thus of no importance, and only the variability in the weight contributes to the aleatory aspect of the phenomenon.
- For very long influence lines, many trucks are simultaneously present on the
structure. The structure is thus almost always loaded, and stress variations due to single trucks are smeared out due to the presence of a global stress whose variations are small and slow.

3.3. Influence of traffic parameters on damage

The results discussed in the previous sections, were computed using the traffic parameters proposed in Baptista (2016). We list them hereafter:

- Free-flow traffic speed, \( v_f \): 80km/h
- Free-flow traffic (Gamma distribution) mean distance, \( d_0 \): 120m
- Free-flow traffic (Gamma distribution) modal distance, \( d_m \): 30m
- Jammed traffic speed: 20 km/h
- Jammed traffic (Beta distribution) minimum distance: 1.33m
- Jammed traffic (Beta distribution) maximum distance: 37m

The distributions corresponding to these parameters are those depicted in Figure 1 and Figure 2, respectively.

Let us now consider the output of a one-day traffic generation, run with these parameters:

- Number of trucks: 8290 (nominal: 8000)
- Total distance \( \sum_{i=1}^{N_{\text{trucks}}} d_i \): 2646km
- Travel time \( \sum_{i=1}^{N_{\text{trucks}}} \frac{d_i}{v_i} \): ~36h (nominal: 18h)

The total travel time (36h), computed a posteriori by dividing the inter-distances by the speed characterizing the corresponding traffic phase, is twice the nominal observation window (18h). An incoherence thus exists between the hypotheses and the results.

Different possibilities could explain this:

- The number of trucks is wrong
- The flux is wrong
- The speeds are wrong
- The inter-distances are wrong

The first two hypotheses derive from real-life measurements. We would thus tend to consider them correct. This leaves us with speeds and distances. Inter-distances play the most important role (in particular, because speeds much beyond 80 km/h would be unrealistic for a truck).

In the literature (e.g., Obrien, Nowak, and Caprani (2022) and Nesterova (2019)) different propositions are formulated for inter-distances,
both in terms of distributions and parameter values. In this work, we keep the same distributions as Baptista (2016), but we propose different parameters. We focus only on the Gamma distribution, since it characterizes the free-flow traffic, which is the most frequent during the day. In particular, we consider a mean distance between vehicles of 60m in free-flow traffic (instead of 120m, Figure 1). The new distribution is thus narrower than the one proposed in Baptista (2016) (see Figure 10).

![Gamma distribution, α = 0, α=2, β=30](image)

Figure 10: Gamma distribution with modified free-flow traffic parameters.

In addition, we also propose to consider a free-flow traffic speed of 90km/h for trucks (cruise control).

With this new set of parameters, the outcome of a one-day traffic generation is the following:
- Number of trucks: 8150 (nominal: 8000)
- Total distance: 1480km
- Travel time: 17h48’ (nominal: 18h)

Consistency between total travel time and the imposed observation window is recovered.

A comparison between the traffic parameters employed in Baptista (2016) and those proposed in this work, in terms of total damage over 250 days is presented in Figure 11(a), for different span lengths. The two sets of parameters lead to comparable damages for span lengths up to ~50m. For longer spans, the difference becomes more important and grows with the influence line length. For a 200m span, the difference is nearly 15% (Figure 11(b)). On the one hand, it seems reasonable that differences, if present, appear for longer spans. In fact, for shorter spans inter-distances play almost no role since damage is mostly due to local effects related to the passage of one truck at a time. On the other hand, for longer spans differences are not negligible and a good choice of the traffic parameters must be sought.

![Damage over 250 days](image)

(a)

![Damage sensitivity to traffic parameters](image)

(b)

Figure 11: Damage sensitivity to traffic parameters.

4. TOWARDS MORE COMPLEX TRAFFIC GENERATION

The approach presented above concerns the traffic on one lane only. In real bridges, of course, traffic may take place on multiple lanes and overtaking can occur. Moreover, uncertainty can also concern trucks transversal deviation from the center of the lane. For instance, the Eurocode EC (2003) requires five different transversal positions to be considered, each one associated to a
different probability of occurrence, in a deterministic way.

The approach presented in this work can be employed almost ‘as is’ to consider such more complex traffic configurations. Indeed, each different lane or transversal position within a lane simply originates a different unitary influence line. For each one of these lines, a different aleatory load train can be generated, in the same way it has been done for the one-lane example proposed in previous sections. By doing so, possible overtaking is naturally included in the model in a non-deterministic way. The only quantity that must be adapted for each line is the number of trucks, since only a portion of the total amount will circulate on a given lane, or at a given transversal position within a lane. Finally, one only needs to apply the different load trains on the corresponding influence lines and superpose all the signals to recover the full stress history at the wanted verification point.

As a side note, if influence surfaces are used instead of lines, it would even be possible to consider transversal deviations as a fully aleatory quantity.

5. CONCLUSIONS
In this work we presented a method to estimate fatigue damage on steel bridges, considering aleatory traffic loads. Synthetic traffic loads are generated numerically and then applied on a finite element model of the structure to deduce aleatory stress histories, following a Montecarlo approach. The statistical model used for the traffic is the one proposed in Baptista (2016). The computation of the stress history is based on the use of unitary influence lines, instead of full finite element runs. This allows to drastically reduce the computational time and brings an important gain in terms of flexibility.

We presented some results obtained using a toy sinusoidal influence line. Even though far from a real-life situation, this simple example allowed us to formulate some important, though preliminary, remarks. It has been observed that a minimum of ten days of traffic should be simulated to achieve a satisfactory Montecarlo convergence. To ensure consistency between travel time and the imposed observation window, we propose to use different parameters than those of Baptista (2016), in particular a mean distance of 60m in free-fluid traffic (instead of 120m), as well as a trucks speed of 90km/h (instead of 80km/h). Comparison of the damage obtained with the two different sets of parameters showed that for short influence lines (up to ~50m) no meaningful difference exists. On the contrary, differences of up to ~15% have been observed for longer influence lines (up to 200m).

Thanks to the influence lines approach, the method can be easily extended to more complex traffic situations (multiple lanes, overtaking, transversal deviation, multiple axles...).

The authors have already applied the method for the study of some real-life structures. However, sufficiently general conclusions could not yet be drawn. In the future, the final goal is to understand how accounting for fully aleatory traffic loads (under different hypotheses) influences the fatigue design of steel bridges, especially compared to the simplified load models proposed in the Eurocode EC (2003) or to an advanced deterministic model.

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6. REFERENCES