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# Dynamic Reliability-Based Design Optimization using Quantum-Inspired Algorithm and Probability Density Evolution Method Assisted by Change of Probability Measure Strategy

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ABSTRACT: Dynamic reliability-based design optimization (DRBDO) is a promising design optimization approach for dynamic structures, when inherent uncertainties are involved in structural parameters and external excitations. This study presents an effective quantum-inspired method to solve the DRBDO problems. In particular, the probability density evolution method (PDEM) is employed for dynamic reliability assessment and the quantum particle swarm optimization (QPSO) algorithm is adopted to solve the optimization problems. To further enhance the efficiency in assessing structural dynamic reliability, the change of probability measure (COM) strategy is incorporated into the PDEM to reuse the information during the optimization process. With this strategy, the number of structural analyses can be significantly reduced. A numerical example concerning the optimization of a nonlinear frame is conducted to demonstrate the effectiveness and efficiency of the proposed method.

## 1. INTRODUCTION

Structural optimization is a powerful technique for achieving economical designs that can realize specified requirements and functions. However, these designs may be unreliable in practice due to uncertainties involved in material properties, construction processes, and external environments (Li and Chen, 2009). To account for the effects of the uncertainties in structural performance, reliability-based design optimization (RBDO) has been extensively studied (Aoues and Chateauneuf, 2010).

RBDO problems are inherently complex to solve because of the need to consider structural reliability (Valdebenito and Schuëller, 2010). Moreover, dynamic reliability-based design optimization (DRBDO) problems pose further computational challenges, as the stochastic response analysis of dynamic systems generally involves high-dimensional integrations and nonlinearities. Therefore, an efficient dynamic reliability analysis method is of great importance. In this regard, the probability density evolution method (PDEM) (Li and Chen, 2004) — demonstrated as a theoretically rigorous and computationally efficient method for structural stochastic response analysis — can be a promising candidate for mitigating the challenges. A recent study conducted by Chen et al. (2020) highlights the significant advantages of using the PDEM in design optimization.

In addition to dynamic reliability analysis methods, the efficiency in solving the DRBDO problems can be greatly affected by the choice of optimization algorithms. Metaheuristic approaches, such as the particle swarm optimization (PSO) algorithm (Kennedy and Eberhart, 1995), are renowned for their versatility and parallelization capacity. While these approaches have been applied to the RBDO of static structures in recent years (Meng et al., 2021).

they suffer from certain limitations such as premature convergence and low convergence speeds. To improve the performance of classical metaheuristic approaches, novel quantum-inspired metaheuristics have been proposed. These algorithms incorporate principles from quantum computing and quantum mechanics theories to enhance the exploration and exploitation capabilities of the optimization algorithm (Li et al., 2020). Among them, the quantum-inspired particle swarm (QPSO) algorithm (Sun et al., 2004) has been successfully applied to various problems except for the optimization of structures under uncertainties.

In the present paper, the QPSO combined with the PDEM is adopted to solve the DRBDO problems. To further enhance the efficiency in assessing structural dynamic reliability, the change of probability measure (COM) (Chen and Wan, 2019), a strategy of reusing the representative points in the framework of the PDEM, is employed during the optimization process. The paper is structured as follows: Section 2 provides the formulation of DRBDO problems. Then the PDEM and the QPSO are introduced in Section 3. The proposed method is also elaborated on in this section. Section 4 demonstrates the performance of the proposed method on optimizing a nonlinear frame structure. Finally, the paper concludes with some conclusions and potential research directions.

## 2. PROBLEM FORMULATION

In general, the DRBDO problems can be formulated as

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & r_j(\boldsymbol{x}) \leqslant 0 \qquad j=1,\cdots,N_r \\ & g_k(\boldsymbol{x}) \leqslant 0 \qquad k=1,\cdots,N_g \\ & \boldsymbol{x} \in X \subset \mathbb{R}^{N_x} \end{aligned} \tag{1}$$

where $\boldsymbol{x}=(x_1,x_2,\cdots,x_{N_x})^{\mathrm{T}}$ is the design vector constrained by the set $X$ ; $f(\boldsymbol{x})$ denotes the objective function; $g_k(\boldsymbol{x})$ represents the $k$ -th standard constraint; $r_j(\boldsymbol{x})$ is the $j$ -th reliability constraint; $N_x$ , $N_g$ and $N_r$ are the numbers of the design variables , the standard constraints and the reliability constraints, respectively.

The objective function and the standard constraints are commonly determined by structural performance and design requirements, such as structural stiffness and construction cost. In this paper, the reliability constraints are defined in terms of the first-passage probability, a typical dynamic reliability measure. Therefore, the reliability constraint functions can be written as

$$r_j(\boldsymbol{x}) = P_{\mathrm{F}_j}(\boldsymbol{x}) - P_{\mathrm{F}_j}^{\mathrm{th}} \leqslant 0, (j=1,\cdots,N_r) \tag{2}$$

where $P_{\mathrm{F}_j}^{\mathrm{th}}$ is the threshold of the failure probability, and $P_{\mathrm{F}_j}(\boldsymbol{x})$ is the failure probability evaluated at the design $\boldsymbol{x}$ for the failure event $F_j$ . Moreover, it is assumed that all the design variables are the mean values of a portion of the random variables.

## 3. METHODOLOGY

To establish the groundwork for the proposed DRBDO method, the section begins by introducing the fundamentals of the PDEM (Li and Chen, 2004) and the PDEM-based method for assessing the first-passage probability (Chen and Li, 2005). Next, the section briefly describes the QPSO algorithm (Sun et al., 2004). Finally, the proposed DRBDO method is elaborated on, highlighting its main features and advantages.

### *3.1. Dynamic reliability assessment with the PDEM*

Consider a multiple-degree-of-freedom stochastic dynamical system whose equation of motion reads

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{\Theta};\boldsymbol{x})\ddot{\boldsymbol{Y}}(t) + \boldsymbol{C}(\boldsymbol{\Theta};\boldsymbol{x})\dot{\boldsymbol{Y}}(t) \\ &+\boldsymbol{F}(\boldsymbol{\Theta},\boldsymbol{Y}(t);\boldsymbol{x}) = \boldsymbol{\Gamma}\xi(\boldsymbol{\Theta},t) \end{aligned} \tag{3}$$

where $\ddot{\boldsymbol{Y}}(t)$ , $\dot{\boldsymbol{Y}}(t)$ and $\boldsymbol{Y}(t)$ are the $N_d$ -dimensional vectors of acceleration, velocity, and displacement of the system, respectively; $\boldsymbol{M}$ , $\boldsymbol{C}$ are the $N_d \times N_d$ mass and damping matrices, respectively; $\boldsymbol{F}$ is the $N_d$ -dimensional linear or nonlinear force vector; $\boldsymbol{\Gamma}$ denotes the $N_d \times N_s$ loading influence matrix; $\boldsymbol{\xi}$ is the $N_s$ -dimensional vector of stochastic excitations; $\boldsymbol{x}$ is the design vector; and $\boldsymbol{\Theta} = (\Theta_1, \cdots, \Theta_{N_\Theta})^{\mathrm{T}}$ is the vector of random variables that embraces all the basic random variables involved in structural parameters and external excitations. The joint probability density function (PDF) of $\boldsymbol{\Theta}$ is denoted by $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$ with $\boldsymbol{\theta}$ being a realization of $\boldsymbol{\Theta}$ .

For a well-posed system, the solution of Eq. (3) exists uniquely, and can serve to calculate the responses of interest. For simplicity, denote the responses by $\boldsymbol{Z}(\boldsymbol{\Theta}, t; \boldsymbol{x}) = (Z_1, \cdots, Z_{N_z})^{\mathrm{T}}$ , where $Z_i$ is the $i$ -th response of the system. Note that the augmented system $(\boldsymbol{Z}, \boldsymbol{\Theta})$ is probability-preserved. According to the principle of preservation of probability (Chen and Li, 2009; Li and Chen, 2008), the evolution of the joint PDF of $\boldsymbol{Z}$ and $\boldsymbol{\Theta}$ is governed by the generalized density evolution equation (GDEE). If only one response of the system $Z$ is of interest, the GDEE is therefore a one-dimensional partial differential equation, namely

$$\frac{\partial p_{Z\boldsymbol{\Theta}}(z, \boldsymbol{\theta}, t; \boldsymbol{x})}{\partial t} + \dot{Z}(\boldsymbol{\theta}, t; \boldsymbol{x}) \frac{\partial p_{Z\boldsymbol{\Theta}}(z, \boldsymbol{\theta}, t; \boldsymbol{x})}{\partial z} = 0 \quad (4)$$

where $p_{Z\boldsymbol{\Theta}}(z, \boldsymbol{\theta}, t; \boldsymbol{x})$ is the joint PDF of $Z$ and $\boldsymbol{\Theta}$ at time $t$ and design $\boldsymbol{x}$ , and $\dot{Z}(\boldsymbol{\theta}, t; \boldsymbol{x})$ is the velocity process of $Z$ in the case $\boldsymbol{\Theta} = \boldsymbol{\theta}$ . The initial condition of Eq. (4) is given by

$$p_{Z\boldsymbol{\Theta}}(z, \boldsymbol{\theta}, t; \boldsymbol{x})\big|_{t=0} = \delta(z - z_0) p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}; \boldsymbol{x}) \quad (5)$$

where $\delta(\cdot)$ is Dirac's delta function, and $z_0$ is the initial value of $Z$ .

To assess the first-passage probability of the system, the PDEM should be incorporated into either the absorbing boundary condition approach (Li and Chen, 2005) or the extreme value distribution approach (Chen and Li, 2005). The latter is adopted.

In accordance with the extreme value distribution approach, denote the equivalent extreme value of the responses of the system over time interval $t \in [0, T]$ as (the symmetric double boundary is employed as an example)

$$Z_{\mathrm{ext}}(\boldsymbol{\Theta}; \boldsymbol{x}) = \max_{t \in [0,T]} \left\{ \max_{i=1,\cdots,N_z} \left[ | Z_i(\boldsymbol{\Theta}, t; \boldsymbol{x}) | / z_i^{\mathrm{th}} \right] \right\} \quad (6)$$

where $z_i^{\mathrm{th}} (i = 1, \cdots, N_z)$ is the threshold for the response $Z_i$ .

Then, construct a virtual stochastic process related to the equivalent extreme value in Eq. (6)

$$W(\tau; \boldsymbol{x}) = Z_{\mathrm{ext}}(\boldsymbol{\Theta}; \boldsymbol{x}) \cdot \sin(2.5\pi \cdot \tau) \quad (7)$$

which satisfies the conditions

$$W(\tau; \boldsymbol{x})\big|_{\tau=0} = 0 \quad (8)$$

$$W(\tau; \boldsymbol{x})\big|_{\tau=\tau_{\mathrm{c}}} = Z_{\mathrm{ext}}(\boldsymbol{\Theta}; \boldsymbol{x}) \quad (9)$$

where $\tau$ is the virtual time, and $\tau_{\mathrm{c}} = 1$ .

Afterwards, obtain the instantaneous PDF of the virtual stochastic process through the PDEM. Specially, by solving the GDEE of the virtual stochastic process, namely

$$\frac{\partial p_{W\boldsymbol{\Theta}}(w, \boldsymbol{\theta}, \tau; \boldsymbol{x})}{\partial \tau} + \dot{W}(\boldsymbol{\theta}, \tau; \boldsymbol{x}) \frac{\partial p_{W\boldsymbol{\Theta}}(w, \boldsymbol{\theta}, \tau; \boldsymbol{x})}{\partial w} = 0 \quad (10)$$

with the initial condition

$$p_{W\boldsymbol{\Theta}}(w, \boldsymbol{\theta}, \tau; \boldsymbol{x})\big|_{\tau=0} = \delta(w) p_{\boldsymbol{\Theta}}(\boldsymbol{\theta}; \boldsymbol{x}) \quad (11)$$

one can get the joint PDF $p_{W\boldsymbol{\Theta}}(w, \boldsymbol{\theta}, \tau; \boldsymbol{x})$ of $W$ and $\boldsymbol{\Theta}$ . Therefore, the PDF of $W(\tau; \boldsymbol{x})$ yields

$$p_W(w, \tau; \boldsymbol{x}) = \int_{\Omega_{\boldsymbol{\Theta}}} p_{W\boldsymbol{\Theta}}(w, \boldsymbol{\theta}, \tau; \boldsymbol{x}) \mathrm{d}\boldsymbol{\theta} \quad (12)$$

According to Eq. (9), the PDF of the equivalent extreme value $Z_{\mathrm{ext}}(\boldsymbol{\Theta}; \boldsymbol{x})$ is

$$p_{Z_{\mathrm{ext}}}(z_{\mathrm{ext}}; \boldsymbol{x}) = p_W(w = z_{\mathrm{ext}}, \tau; \boldsymbol{x})\big|_{\tau=\tau_{\mathrm{c}}} \quad (13)$$

Finally, calculate the first-passage probability by a one-dimensional integral, that is

$$P_{\mathrm{F}}(\boldsymbol{x}) = \int_1^{\infty} p_{Z_{\mathrm{ext}}}(z_{\mathrm{ext}}; \boldsymbol{x}) \mathrm{d}z_{\mathrm{ext}} \quad (14)$$

For general dynamical systems, the GDEE is solved by numerical methods, in which the finite difference method is adopted herein. The numerical procedures are outlined as follows:

1. Select representative points $M=\{\boldsymbol{\theta}_q\}_{q=1}^{N_{\text{sel}}}$ in the domain $\Omega_{\boldsymbol{\Theta}}$ with the generalized F-discrepancy-based point selection strategy (Chen et al., 2016) and calculate the corresponding assigned probabilities; $N_{\text{sel}}$ is the number of the selected points.
2. Carry out deterministic structural analyses for all the representative points to obtain the values of the equivalent extreme value $\{Z_{\text{ext}}(\boldsymbol{\theta}_q;\boldsymbol{x})\}_{q=1}^{N_{\text{sel}}}$.
3. Solve the GDEEs in terms of the equivalent extreme value $\{Z_{\text{ext}}(\boldsymbol{\theta}_q;\boldsymbol{x})\}_{q=1}^{N_{\text{sel}}}$ with the finite difference method, and then perform numerical integration over $\Omega_{\boldsymbol{\Theta}}$, yielding the numerical solution $p_{Z_{\text{ext}}}(z_{\text{ext}};\boldsymbol{x})$.
4. Calculate the first-passage probability using Eq. (14).

### *3.2. Quantum-inspired particle swarm optimization algorithm*

Sun et al. (2004) proposed the QPSO algorithm, drawing inspiration from the trajectory analysis of the canonical PSO and quantum mechanics theories. In quantum mechanics, the state of a physical particle, such as position or momentum, is characterized by a wave function $\psi(x)$, which is essential to determine the probabilities of measurement outcomes for the quantum system (Griffith and Schroeter, 2018).

Given this, the QPSO assumes that the position of a particle is described by the wave function $\psi(x)$ of a physical particle in a Delta potential well. According to the statistical significance of the wave function, the PDF of the particle's position is

$$|\psi(x)|^2=\frac{1}{L}e^{-\frac{2|x-p|}{L}} \tag{15}$$

where $L$ is the characteristic length of the Delta potential well, and $p$ is the attractor of the particle's position.

By employing the Monte Carlo Simulation method to simulate the process for quantum measurement, one can obtain the position of the particle, that is

$$x_{i,t+1}^j=p_{i,t}^j\pm\frac{L_{i,t}^j}{2}\ln(1/u_{i,t}^j) \tag{16}$$

where $x_{i,t+1}^j$ is the $j$-th component of the $i$-th particle's position at $(t+1)$-th iteration; $u_{i,t}^j$ is a random number uniformly distributed within the interval $[0,1]$; $p_{i,t}^j$ is the $j$-th component of the attractor defined by

$$p_{i,t}^j=\frac{\varphi_{i,t}^j\text{pbest}_{i,t}^j+\phi_{i,t}^j\text{gbest}_t^j}{\varphi_{i,t}^j+\phi_{i,t}^j} \tag{17}$$

where $\varphi_{i,t}^j$ and $\phi_{i,t}^j$ are random numbers satisfying uniform distribution within the over $[0,1]$; $\text{pbest}_i$ is the optimal position of the $i$-th particle; gbest is the global optimal position of the particle swarm. The characteristic length $L_{i,t}^j$ is determined by

$$L_{i,t}^j=2\alpha\cdot|x_{i,t}^j-p_{i,t}^j| \tag{18}$$

where $\alpha$ is the contraction-expansion coefficient set to 1/0.96.

### *3.3. Proposed DRBDO method*

Despite the remarkable efficiency of the PDEM in assessing dynamic reliability and the commendable convergence performance of the QPSO, the DRBDO for general structures remains arduous. The design optimization procedure necessitates repeated dynamic reliability analyses, which can be time-consuming and resource-intensive. To address these issues, the change of probability measure (COM) is incorporated into the crude PDEM-QPSO-based DRBDO method.

The COM is first proposed to quantify simultaneously aleatory and epistemic uncertainty in the framework of the PDEM. It has been applied to assess the time-variant reliability of structures with monotonically deteriorating materials (Wan et al., 2020) and optimize frame structures based on the sequential approximate programming technique (Chen et al., 2020). Since the COM strategy can reuse the information of the representative points during the optimization process, it leads to significant computational savings.

For the crude PDEM-QPSO-based DRBDO method, the PDEM is adopted repeatedly to assess the dynamic reliability for each updated design. Denote all the designs and the corresponding representative point sets generated during the initial $k$ iterations by $\mathcal{X}=\left\{\boldsymbol{x}_1,\cdots,\boldsymbol{x}_{N_{pd}}\right\}$ and $\mathcal{M}=\left\{M_1,\cdots,M_{N_{pd}}\right\}$, respectively; $\boldsymbol{x}_i$ and $M_i$ denote the $i$ -th design vector and the corresponding representative point set, respectively. Due to the design variables being the mean values of the random variables, the changes in the design variables can affect the joint PDF of the random variables. Assume that a round of reliability analysis has been performed at design $\boldsymbol{x}_k$ through the PDEM with the representative point set $M_k$. If $M_k$ remains constant at design $\boldsymbol{x}_{k+1}$, the assigned probabilities of the representative points in $M_k$ will therefore vary from $P_k=\left\{P_{\boldsymbol{\theta}_1,k},\cdots,P_{\boldsymbol{\theta}_{N_{rp}},k}\right\}$ to $P_{k+1}=\left\{P_{\boldsymbol{\theta}_1,k+1},\cdots,P_{\boldsymbol{\theta}_{N_{rp}},k+1}\right\}$, resulting in changes in the probability measures of the representative points in $M_k$; $P_{\boldsymbol{\theta}_i,k}$ denotes the assigned probability of the representative point $\boldsymbol{\theta}_q$ in $M_k$. Because the representative points are unchanged, no additional deterministic analyses of the structural response are required to assess the first-passage probability of the design $\boldsymbol{x}_{k+1}$. This means that only the assigned probabilities of the representative points need to be re-calculated at a new design when the COM strategy is employed.

To reduce the numerical errors in dynamic reliability analyses caused by the numerical solution of the PDEM-COM strategy, the following constraint on an updated design should be satisfied if the COM strategy is adopted:

$$\begin{gathered} d_{k+1}\le d_{\lim} \\ d_{k+1}=\min\|\boldsymbol{x}_{k+1}-\boldsymbol{x}_{PDEM}\|_\infty,\text{for all }\boldsymbol{x}_{PDEM}\in\mathcal{X} \end{gathered} \tag{19}$$

where $\boldsymbol{x}_{k+1}$ is the $(k+1)$ -th design vector generated during the optimization process, and $\boldsymbol{x}_{PDEM}$ is the design vector whose failure probability has been evaluated with the PDEM.

To deal with the constraints, the penalty-based method is adopted. With this method, the original constrained optimization problem Eq. (1) is transformed to the following unconstrained optimization problem:

$$\min_{\boldsymbol{x}}\quad \tilde{f}(\boldsymbol{x})=f(\boldsymbol{x})\cdot[1+\beta P(\boldsymbol{x})] \tag{20}$$

where $\beta=100\cdot n/N_{\text{it}}$, $n$ is the current iteration number, and $N_{\text{it}}$ is the maximum iteration number. The penalty function $P(\boldsymbol{x})$ is defined by

$$\begin{gathered} P(\boldsymbol{x})=\sum_i c_i(\boldsymbol{x}) \\ \begin{cases} c_i(\boldsymbol{x})=\max[0,10\times g_i(\boldsymbol{x})] \\ \text{or} \\ c_i(\boldsymbol{x})=\max[0,r_j(\boldsymbol{x})/P_{\text{F}_j}^{\text{th}}] \end{cases} \end{gathered} \tag{21}$$

In summary, the procedure of the proposed DRBDO method, namely the PDEM-COM-QPSO-based method, includes the following steps:

1. Initialization. Initialize the PDEM solver and the QPSO optimizer. Evaluate the first-passage probabilities of the all designs at the initial iteration using the PDEM, and store the representative point sets along with their corresponding dynamic responses. Set the iteration index $n=0$.
2. Update of the design variables. Calculate the fitness values of all the designs (Eq. (20)) at the $n$ -th iteration, and update the design variables according to Eq. (16)-(18). Set $n=n+1$.
3. Reliability analysis. For each new design $\boldsymbol{x}$ generated at the $n$-th iteration, use the PDEM-COM to evaluate the first-passage probability of $\boldsymbol{x}$ if the condition Eq. (19) is satisfied; Otherwise, use the PDEM instead. If the PDEM is adopted, store the representative point set $M$ related to $\boldsymbol{x}$ along with the corresponding dynamic responses.
4. Repeat or end. Repeat Step 2 ~ Step3, and set $n=n+1$ until the maximum iteration number $N_{\text{it}}$ is hit.

## 4. APPLICATION

This section presents the DRBDO of a two-storied nonlinear frame structure to demonstrate the effectiveness and efficiency of the proposed method. The structure is shown in Figure 1. The floor height of the structure is $h = 3.6\text{m}$. The lumped masses for the first and the second floors are $1.8\times10^5\text{kg}$ and $1.2\times10^5\text{kg}$, respectively. The nonlinearity of the structure is characterized by the Bouc-Wen model (Ma et al., 2004) whose parameters are set as: $\alpha = 0.04$, $A = 1$, $\beta = 320$, $\gamma = 150$, $n = 1$, $d_\nu = 300$, $d_\eta = 300$, $p = 1000$, $q = 0.25$, $\psi = 0.05$, $\lambda = 0.5$, $d_\psi = 5$, and $\zeta_s = 0.99$. The modal damping ratios are 0.05.

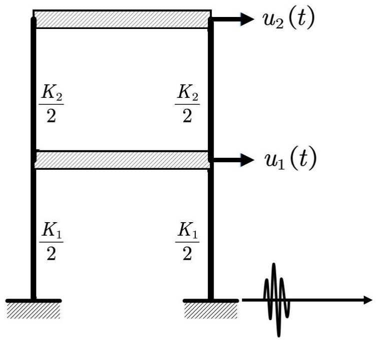


*Figure 1: A 2-storey frame structure.*

*Table 1: Distribution parameters of the random variables.*

| *Random variable* | *Type of distribution* | *Mean value* | *Coefficient of variation* |
|---|---|---|---|
| $K_1$ | *Normal* | $x_1$ | *0.05* |
| $K_2$ | *Normal* | $x_2$ | *0.05* |
| $A_1$ | *Normal* | *0.10g* | *0.10* |
| $A_2$ | *Normal* | *0.10g* | *0.10* |

In this example, the lateral inter-story stiffnesses of the structure, i.e. $\boldsymbol{K} = (K_1, K_2)^{\mathrm{T}}$, are assumed to be random variables following independent normal distributions. The means of the lateral stiffnesses are taken as design variables, and denoted by $\boldsymbol{x} = (x_1, x_2)^{\mathrm{T}}$. Besides, the structure is subjected to random earthquake excitations, which are random combinations of the El Centro acceleration records in N-S and E-W directions, namely $\ddot{u}_{\mathrm{g,NS}}(t)$ and $\ddot{u}_{\mathrm{g,EW}}(t)$. The random earthquake excitation is formulated as

$$\ddot{u}_g(t) = A_1\ddot{u}_{\mathrm{g,NS}}(t) + A_2\ddot{u}_{\mathrm{g,EW}}(t) \tag{22}$$

where $A_1$ and $A_2$ are the random combination coefficients. The distribution parameters of all the random variables are presented in Table 1, in which $g$ is the gravity acceleration, taking the value $9.807\text{m}/\text{s}^2$.

The design optimization aims at minimizing the sum of the lateral inter-story stiffnesses. In order to improve overall stability of the structure, it is required that the lateral stiffness at the lower level should be greater than that at the upper level. Moreover, the failure probability of the structure should be lower than the threshold $P_{\mathrm{F}_j}^{\mathrm{th}} = 0.01$. Consequently, the optimization problem is defined by

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & f(\boldsymbol{x}) = x_1 + x_2 \\ \text{s.t.} \quad & x_2 - x_1 \le 0 \\ & P_{\mathrm{F}}(\boldsymbol{x}) - P_{\mathrm{F}}^{\mathrm{th}} < 0 \\ & 0.5 \le x_1 \le 1.2,\ 0.3 \le x_2 \le 1.0 \end{aligned} \tag{23}$$

where the failure probability $P_{\mathrm{F}}(\boldsymbol{x})$ is defined by

$$P_{\mathrm{F}}(\boldsymbol{x}) = \Pr\left\{\max_{r=1,2}\left[\max_{t\in[0,T]}\left[\left|U_r(\boldsymbol{\Theta}, t; \boldsymbol{x})\right| / u_r^{\mathrm{th}}\right]\right] > 1\right\} \tag{24}$$

where $U_r$ is the inter-story drift of the $r$-th floor, and $u_r^{\mathrm{th}} = h/250$ is the limit of the inter-story drift.

The optimization in Eq. (23) is solved by three methods, namely the crude PDEM-PSO method (Method 1), the crude PDEM-QPSO method (Method 2), and the proposed method (Method 3). For the optimization algorithm, the number of the population size is 20, and the maximum number of the iteration is 300. For the PDEM, 300 representative points are selected. The parameter $d_{\lim}$ in Eq. (19) is $0.01\times10^8\,\text{N}\cdot\text{m}$.

Figure 2 presents the iteration history of the objective function. It can be observed that all the three methods converge rapidly to nearly identical solutions, although Method 3 converges at a slightly slower rate than Method 1 and 2 during the early stages of optimization. The final objective function values, the final designs, and the corresponding failure probabilities are shown in Table 2, where $N_{PF}$ is the number of the reliability analyses performed by the PDEM (without the COM strategy); the units of the objective function values and the design variables are $\times 10^8\,\mathrm{N}\cdot\mathrm{m}^{-1}$. It is shown that the proposed method can considerably reduce the computational costs by approximately 90% without sacrificing accuracy. The relative error in the final objective function values is less than 0.0039. Furthermore, the failure probabilities of the final designs generated by all the methods reach the 0.01 threshold, underscoring the importance of considering reliability constraints in the design optimization process.

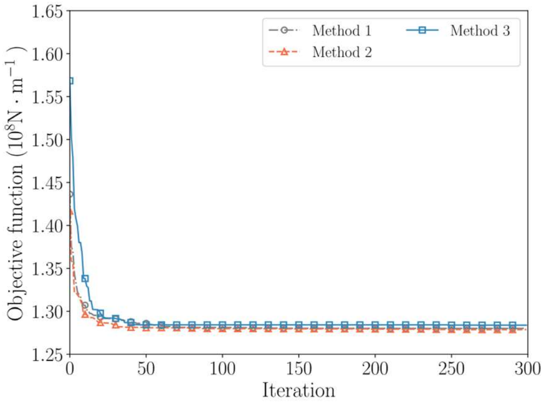


*Figure 2: Iteration history in terms of the objective function value.*

*Table 2: The Final objective function values, design variables, and failure probabilities.*

| Method | $N_{PF}$ | Objective function value | $x_1$ | $x_2$ | $P_F$ |
|---|---|---|---|---|---|
| *1* | *6000* | *1.281* | *0.867* | *0.414* | *0.01* |
| *2* | *6000* | *1.279* | *0.867* | *0.412* | *0.01* |
| *3* | *633* | *1.284* | *0.869* | *0.415* | *0.01* |

## 5. CONCLUSIONS

This study presents an effective method for solving a specific type of DRBDO problems, where the design variables are the distribution parameters of the random variables. Specifically, the proposed method utilizes the QPSO to solve the optimization problems, while employing the PDEM to evaluate structural dynamic reliability. Moreover, the COM strategy is integrated into the PDEM to take fully advantage of the information associated with the representative points. Finally, the effectiveness and efficiency of the proposed method is demonstrated by a numerical example of the optimization of a nonlinear frame structure.

In future research, it is essential to further test and enhance efficacy of the proposed method. Applying the proposed method to optimize high-dimensional structures is also a promising direction of research.

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