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Random vibration and global dynamic analyses of nonlinear MDOF systems under combined additive and multiplicative excitation

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ABSTRACT: The random vibration and global dynamic analyses of structures subjected to combined additive and multiplicative excitation are challenging issues. In this paper, the direct probability integral method (DPIM), as a novel method, is developed to address these issues. The first aim of the paper is to investigate the random vibration responses of multi-degree-of-freedom (MDOF) systems under combined excitation by DPIM, showing the evolutionary process of probability density functions of the response. The second aim is to illustrate the global dynamics of MDOF systems under combined excitation. By incorporating the ε-committor functions, a DPIM-based strategy is proposed to obtain the generalized stochastic basins of attractions of system and evaluate the corresponding stochastic basin stability through global integrity measure (GIM). In the numerical example, the stochastic dynamic analysis of nonlinear MDOF coupled ship model under random oblique wave is performed, demonstrating the accuracy and efficiency of DPIM by comparing with Monte Carlo simulation (MCS) method. It is also indicated that the safety basin of ship can be effectively represented in a probabilistic way, and the erosion of stochastic safety basin will be appeared with increase of significant wave height.

1. INTRODUCTION

For the stochastic dynamic analysis, most of researches focuses on the dynamic behaviors of systems under the additive or multiplicative excitation. Nevertheless, in practice, the situation that systems excited by combined additive and multiplicative excitation is also inevitable (Chen and Sun 2020), e.g., rolling motion of ship under the oblique wave (Umeda et al. 2016). Therefore, the study of the dynamical properties of nonlinear multi-degree-of-freedom (MDOF) systems under combined additive and multiplicative excitation is very significant and meaningful, facilitating to improve structural security.

The randomness of dynamic systems is stated by the probability density function (PDF) or the moments. In the random vibration analysis, to achieve the response PDF of system, several methods were developed, such as path integral method (Di Paola, 2020), probability density evolution method (Li and Chen 2009) and cell mapping method (Yue et al. 2019). However, the random vibration analysis of nonlinear MDOF systems under combined additive and multiplicative excitation is insufficient (Chen and Sun 2020).

The global dynamic analysis concentrates on the change of attractor or corresponding basin when the perturbation amplitude exceeds the limitation, which is closely related to the damage to structures. Thus, it plays a key role in various engineering structures (Li et al. 2020). To study the evolution of basin and its stability of stochastic system, ε-committor function (Lindner and Hellmann 2019) is introduced as generalized stochastic basin and global integrity measure (GIM) (Orlando et al. 2019) is adopted as integrity measure to evaluate the basin stability. The committor function has a very simple mathematical description, satisfying the backward Kolmogorov equation (Li et al. 2019). This equation is usually difficult to be solved, especially for MDOF system. Although the Monte Carlo simulation (MCS) method is versatile (Benedetti et al. 2020), it is time-consuming and converges randomly (Misraji et al. 2020). Thus, the calculation of committor function for MDOF stochastic system with high accuracy and efficiency has been a challenge issue.

In this paper, a novel direct probability integral method (DPIM) (Chen and Yang 2019, 2021) is developed to perform the random vibration and global dynamic analyses of nonlinear MDOF systems under combined additive and multiplicative excitation. Firstly, the probability density integral equation (PDIE) of MDOF system under combined excitation is constructed in terms of the perspective of probability conservation. And the response PDF is realized by efficient solving PDIE via DPIM. Then, a new DPIM-based strategy is proposed to achieve an effective estimator of the global dynamic property for nonlinear MDOF system, in which ε-committor function and GIM are obtained from the viewpoint of probability. Finally, to further verify the efficiency and accuracy of proposed method, the random vibration and global dynamic analyses of MDOF coupled ship model subjected to oblique wave are performed, and the stochastic dynamic behavior of ship is also investigated.

2. DIRECT PROBABILITY INTEGRAL METHOD

In this section, the fundamental concept of DPIM is introduced briefly. The PDIE of MDOF system subjected to combined additive and multiplicative excitation is firstly established, and DPIM is then applied to solve PDIE numerically.

Let the motion equation of MDOF system under combined additive and multiplicative excitation be

$$\dot{\mathbf{Y}}(t) = f(\mathbf{Y},t) + \sum_{i=1}^{m} l_i(\mathbf{Y},t)\mathbf{W}_i(\boldsymbol{\theta},t) \qquad (1)$$

where $\mathbf{Y}(t)$ indicates a response vector of dynamic system; $f(\mathbf{Y},t)$ means a deterministic nonlinear function; $\mathbf{W}_i(\boldsymbol{\theta}, t)$ with $i = 1, 2, ..., m$ denotes m Gaussian white noise process vectors; $\boldsymbol{\theta}$ represents the random vector; If $l_i(\mathbf{Y}, t)$ is constant (i.e., $l_i(\mathbf{Y}, t)=l_i$), it means that the dynamic system is acted by additive excitation only; If the function $l_i(\mathbf{Y}, t)$ depends on the response vector $\mathbf{Y}(t)$, the system is considered to be acted by multiplicative excitation.

2.1. PDIE of MDOF systems under combined additive and multiplicative excitation

For MDOF dynamic system with input random vector $\boldsymbol{\Theta}$ and output random vector $\mathbf{Y}$, it satisfies the principle of probability conservation (Li and Chen 2009), which is

$$\int_{\Omega_{\mathbf{Y}}} p_{\mathbf{Y}}(\mathbf{y},t)\mathrm{d}\mathbf{y} = \int_{\Omega_{\boldsymbol{\Theta}}} p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \qquad (2)$$

where $p_{\mathbf{Y}}(\mathbf{y},t)$ means PDF of $\mathbf{y}$ at any time t; $\Omega_{\mathbf{Y}}$ indicates the total sample space of output random vector $\mathbf{Y}$; $p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$ denotes PDF of input random vector $\boldsymbol{\Theta}$; $\Omega_{\boldsymbol{\Theta}}$ denotes the input sample space. Specifically, Eq. (2) indicates that the total probability measure of output equals to that of input and is invariant in the evolution of dynamic system (Chen and Yang 2019).

In Eq. (2), the physical evolution process of output response vector $\mathbf{Y}$ is governed by the motion equation of system, i.e., Eq. (1), which can also be expressed by following deterministic mapping $\mathcal{G}$: $\mathbf{Y}(t)=g(\boldsymbol{\theta},t)$. In order to explicitly describe the randomness propagation from the input random vector $\boldsymbol{\Theta}$ to output response vector $\mathbf{Y}$, PDIE is derived through the deterministic mapping $\mathcal{G}$ and Dirac delta function (Chen and Yang 2019)

$$p_{\mathbf{Y}}(\mathbf{y},t)=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\delta\left[\mathbf{y}-g(\boldsymbol{\theta},t)\right]p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})\,\mathrm{d}\,\boldsymbol{\theta} \qquad (3)$$

which can achieve the joint PDF of output random vector $\mathbf{Y}$. Furthermore, by using the property of Dirac delta function and performing marginal integral, the dimension reduction of PDIE for

MDOF systems is realized and PDF of concerned response $y_\ell(t)$ is then given by

$$p_{Y_\ell}(y_\ell,t)=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\delta\left[y_\ell-g_\ell(\boldsymbol{\theta},t)\right]p_{\boldsymbol{\Theta}}(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta} \quad (4)$$

Equation (4) is termed as PDIE of concerned response $y_\ell(t)$. Since the multiple dimensional integrals with Dirac delta function in Eq. (4), it is hard to solve this equation analytically. To deal with this problem, the numerical process of Eq. (4) is presented in next subsection.

2.2. Numerical procedure for solving probability density integral equation

In this subsection, the numerical solution of PDIE is achieved based on DPIM. Two key techniques are the partition of probability space and the smoothing technique of Dirac delta function (Chen and Yang 2019). The corresponding numerical formula of PDIE is

$$p_{Y_\ell}(y_\ell,t)=\sum_{q=1}^{N}\left\{\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\left[y_\ell-g_\ell(\boldsymbol{\theta}_q,t)\right]^2/2\sigma^2}P_q\right\} \quad (5)$$

where $\boldsymbol{\theta}_q$ represents qth representative point in input sample space; N indicates the total number of representative points; P_q means the assigned probability of qth representative point; $g_\ell(\boldsymbol{\theta}_q, t)$ denotes the dynamic response of qth representative point, obtained by adopting a classical fourth-order Runge-Kutta method in this paper; σ is the smoothing parameter of Dirac delta function, which can be determinated by an adaptive formula based on kernel density estimation (Chen and Yang 2021). More details of the numerical procedure of DPIM are referred to Ref. (Chen and Yang 2021). As illustrated in Eq. (4), DPIM achieves random vibration analysis of MDOF system under combined excitation by solving the deterministic motion equation and PDIE in turn and using a decoupled way.

3. GENERALIZED STOCHASTIC BASIN AND ASSESS ITS STABILITY VIA DPIM

In the research of global dynamic property of MDOF system, the generalized stochastic basin and its stability is a practical and meaningful topic, contributing to improve the operating safety of structure. In this section, it will be achieved via a DPIM-based strategy effectively.

3.1. ε-committor function for studying the basin of stochastic system

In this subsection, the ε-committor function is introduced herein to determine generalized stochastic basin. Firstly, let X_k be a Markov chain, and the probability it has been absorbed into a unique exit state at every time step k is denoted as exit probability ε, in which ε obeys uniform distribution and $\varepsilon\in(0,\ 1]$. Then, assume that $\mathcal{I}_A(x)$=1 if $x\in A$, else $\mathcal{I}_A(x)$=0, and denote that $p^a = \varepsilon\mathcal{I}_A$, $p^b = \varepsilon\mathcal{I}_B = \varepsilon(\mathcal{I}-\mathcal{I}_A)$. Substitute this into backward Kolmogorov equation with Dirichlet boundary conditions (Norris 1998), we have

$$(I-(1-\varepsilon)M)q_\varepsilon(A)=\varepsilon\mathcal{I}_A(x) \quad (6)$$

which the solution $q_\varepsilon(A)$ is the ε-committor function of region A. By utilizing the Neumann inversion formula (Lindner and Hellmann 2019), the unique solution of Eq. (6) is

$$q_\varepsilon(A)=\varepsilon\left[\sum_{k=0}^{\infty}(1-\varepsilon)^k M^k\right]\mathcal{I}_A \quad (7)$$

where $\mathcal{I}_A(x)=:\mathcal{I}_A$ that the argument x is removed while it is clear from the context. Interpreted in terms of probability, ε-committor $q_\varepsilon(A)$ indicates the probability that a given trajectory starting in initial state moves toward region A and stays there with respect to a finite timescale. Specially, in Ref. (Lindner and Hellmann 2019), it has been verified that ε-committor function is a genuine generalization of the notion of stochastic basin of Markov system. For the low-dimensional Markov systems, the finite-time ε-committor function can be calculated by linear Eq. (7). However, the dimensionality of Markov chain increases exponentially with the number of DOFs of system. In next subsection, we will therefore focus on evaluating ε-committor function of MDOF system through its probability interpretation and a novel DPIM-based strategy.

3.2. *ε-committor function and global integrity measure of MDOF system evaluated by DPIM*

In this subsection, the ε-committor function, as a generalized stochastic basin, is evaluated by DPIM efficiently through its probability interpretation. Furthermore, for assessing the stochastic basin stability, GIM is introduced and be evaluated by DPIM from the viewpoint of probability.

The GIM is denoted as the normalized volume of the basin in phase space. Let the phase space of MDOF system be $\Omega_{\mathbf{X}}$ and $\mathbf{x}$ means the point in $\Omega_{\mathbf{X}}$, GIM is expressed as

$$\mathrm{GIM} = \int_{\Omega_{\mathbf{X}}} h_B(\mathbf{x})\mathrm{d}\mathbf{x} \tag{8}$$

where $h_B(\mathbf{x})$ is a predetermined characteristic function, indicating a set $B \subseteq \Omega_{\mathbf{X}}$ that all states in the set B converge to attractor A under non-small perturbations, which is

$$h_B(\mathbf{x}) = \begin{cases} 1, & \lim_{t\to\infty} \eta(\mathbf{x},t) = A, \quad \forall \mathbf{x} \in \Omega_{\mathbf{X}} \\ 0, & \lim_{t\to\infty} \eta(\mathbf{x},t) \neq A, \quad \forall \mathbf{x} \in \Omega_{\mathbf{X}} \end{cases} \tag{9}$$

where η denotes a mapping function of $\mathbf{x}$. Introducing an expectation operator, the mean of $h_B(\mathbf{x})$ is obtained

$$b = \bar{h}_B(\mathbf{x}) = E_f[h_B(\mathbf{x})] = \int_{\Omega_{\mathbf{X}}} h_B(\mathbf{x}) p_{\mathrm{pert}}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{10}$$

in which $p_{\mathrm{pert}}(\mathbf{x})$ means an initial probability distribution of $\mathbf{x}$. Specially, Eq. (10) also indicates the basin stability b of stochastic system（Lindner and Hellmann 2019）. Then, GIM is calculated approximately by

$$\mathrm{GIM} \simeq \overline{\mathrm{GIM}} = \bar{h}_B(\mathbf{x})V_{\Omega_{\mathbf{X}}} = E_f[h_B(\mathbf{x})]V_{\Omega_{\mathbf{X}}} \tag{11}$$

where $V_{\Omega_{\mathbf{X}}}$ indicates the hypervolume of any region $\Omega_{\mathbf{X}}$. It worth to point out that the term $h_B(\mathbf{x})$ in the right hand of Eq. (10) also represents the basin structure of system. As mentioned in Section 3.2, we can utilize ε-committor function to replace $h_B(x)$ in Eq. (10), the generalized basin stability is thus obtained by

$$b_{\mathrm{gen}} = \int_{\Omega_{\mathbf{X}}} q_{\varepsilon}^{A}(\mathbf{x}) p_{pert}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{12}$$

where $q_{\varepsilon}^{A}(\mathbf{x})$ denotes the ε-committor function of attractor A, which can be solved by Monte Carlo integration (Benedetti et al. 2020). Nevertheless, this would be quite expensive, especially for high-dimensional system. Thus, in this paper, DPIM is applied to evaluate ε-committor function efficiently. To achieve this, it is significant to point out that the ε-committor function $q_{\varepsilon}^{A}(\mathbf{x})$ can be interpretated as the probability that $\mathbf{x}$ runs into A within time t starting from initial condition $\mathbf{x}_0$ while we draw t from an appropriate choice distribution ρ_{run} (Lindner and Hellmann 2019). Let the $\rho_{\mathrm{run}}=\varepsilon \mathrm{e}^{-\varepsilon t}$, ε-committor $q_{\varepsilon}^{A}(\mathbf{x})$ is given as follows

$$q_{\varepsilon}^{A}(\mathbf{x}) = \int p(\mathbf{x}(t) \in A | \mathbf{x}(0) = \mathbf{x}_0)\varepsilon \mathrm{e}^{-\varepsilon t}\mathrm{d}t \tag{13}$$

where the term $p(\mathbf{x}(t) \in A | \mathbf{x}(0) = \mathbf{x}_0)$ indicates the probability $\mathbf{x}$ runs to A within time t starting from a given initial condition $\mathbf{x}_0$. Introduce the PDIE of response probability on concerned domain (Chen et al. 2023), we have

$$p(\mathbf{x}(t) \in A | \mathbf{x}(0) = \mathbf{x}_0) = \int_{\Omega_{\Theta}} K(z) p_{\Theta}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \tag{14}$$

in which Ω_{Θ} means the sample space of input random vector $\boldsymbol{\theta}$; $z = \dfrac{\| g(\boldsymbol{\theta}, \mathbf{x}_0, t) - \mathbf{x}_{\mathrm{cen}} \|}{h_d}$; $g(\cdot)$ is a deterministic mapping $\mathcal{G}$: $\mathbf{x}(t) = g(\boldsymbol{\theta},\mathbf{x}_0,t)$; $\mathbf{x}_{\mathrm{cen}}$ represents the centroid of metastable region of attractor A; $K(\cdot)$ denotes the kernel function or Parzen window function. Substituting Eq. (14) into Eq. (13), the ε-committor function is expressed by

$$q_{\varepsilon}^{A}(\mathbf{x}) = \int \int_{\Omega_{\Theta}} K(z) p_{\Theta}(\boldsymbol{\theta})\varepsilon \mathrm{e}^{-\varepsilon t}\mathrm{d}\boldsymbol{\theta}\mathrm{d}t \tag{15}$$

Moreover, to assess the generalized stochastic basin stability by PDIE, inserting Eq. (15) into Eq. (12) yields

$$b_{\mathrm{gen}} = \int \int_{\Omega_{\mathbf{X}_0}} \int_{\Omega_{\Theta}} K(z) p_{\Theta}(\boldsymbol{\theta}) \cdots \cdots p_{pert}(\mathbf{x}_0)\varepsilon \mathrm{e}^{-\varepsilon t}\mathrm{d}\mathbf{x}_0\mathrm{d}\boldsymbol{\theta}\mathrm{d}t \tag{16}$$

in which $\Omega_{\mathbf{X}_0}$ means the sample space of initial perturbation on state $\mathbf{x}_0$. By combining the sample space $\boldsymbol{\Omega}_{\mathbf{X}_0}$ and $\boldsymbol{\Omega}_{\boldsymbol{\Theta}}$, an augmented sample space $\Omega_{\boldsymbol{\Theta}_H}$ ($\Omega_{\boldsymbol{\Theta}_H} = \Omega_{\mathbf{X}_0} \times \Omega_{\boldsymbol{\Theta}}$) is formed, and Eq. (16) then becomes

$$b_{\text{gen}} = \int \int_{\Omega_{\boldsymbol{\Theta}_H}} K(\mathbf{z}) p_{\boldsymbol{\Theta}_H}(\boldsymbol{\theta}_h) \varepsilon \mathrm{e}^{-\varepsilon t} \mathrm{d}\boldsymbol{\theta}_h \mathrm{d}t \quad (17)$$

where $\boldsymbol{\theta}_h = (\mathbf{x}_0, \boldsymbol{\theta})$ denotes an augmented vector. Substituting Eq. (17) into Eq. (11), the evaluation of GIM is achieved by PDIE of response probability from the viewpoint of probability, which is

$$\begin{aligned} \text{GIM} \simeq \overline{\text{GIM}} &= b_{\text{gen}} V_{\Omega_{\mathbf{X}}} \\ &= \int \int_{\Omega_{\boldsymbol{\Theta}_H}} K(z) p_{\boldsymbol{\Theta}_H}(\boldsymbol{\theta}_h) \varepsilon \mathrm{e}^{-\varepsilon t} \mathrm{d}\boldsymbol{\theta}_h \mathrm{d}t V_{\Omega_{\mathbf{X}}} \end{aligned} \quad (18)$$

The detailed numerical procedure for solving ε-committor function and $\overline{\text{GIM}}$ by DPIM can refer to Ref. (Chen et al. 2023).

4. NUMERICAL EXAMPLE

4.1. Forcedly-parametrically excited rolling of nonlinear MDOF coupled ship model under random oblique wave

In the marine engineering field, study roll motion of ship under random oblique wave, and give the stochastic safety basin are the challenge and meaningful research topics (Anastopoulos and Spyrou 2016), especially for MDOF ship system. As a famous numerical method, path integral (PI) method has been extended to investigates the PDF of non-linear random roll motion of SDOF ship system（Zhu and Duan 2016）. However, the main difficult of PI method is that the computational effort increases with power law as the increase of the number of state variables, which is hard be extended to solve MDOF ship system, let alone the global dynamic analysis.

in real word, the ship itself belongs to a nonlinear dynamic system, which its coupling of rolling may be ignored in the SDOF rolling motion equation.

In this example, the random vibration and global dynamic analyses of forcedly-parametrically excited rolling of nonlinear MDOF coupled ship model under random oblique wave are implemented via DPIM, in which the large amplitude rolling motion and the stochastic safe basin of ship are investigated. To better research the ship motion under random oblique wave, a nonlinear four degrees of freedom coupled motion equation (Zhou and Tang 2018) is considered to better simulate the ship motion, i.e.,

$$\begin{cases} \sum M_{xx} = (I_{xx}\dot{\phi}_x - I_{xz}\dot{\phi}_x) - \phi_z\phi_y(I_{yy} - I_{zz}) - I_{xz}\phi_x\phi_y \\ \sum M_{yy} = I_{yy}\dot{\phi}_y - \phi_x\phi_z(I_{zz} - I_{xx}) + I_{zx}\phi_x^2 - I_{zx}\phi_z^2 \\ \sum M_{zz} = (I_{zz}\dot{\phi}_z - I_{zx}\dot{\phi}_x) - \phi_y\phi_x(I_{xx} - I_{yy}) + I_{zx}\phi_z\phi_y \\ \sum F_{oz} = m\dot{v}_z \end{cases} \quad (19)$$

where ϕ_x, ϕ_y, ϕ_z are the angular velocity around X-axis, Y-axis and Z-axis, respectively; $\sum M_{xx}$, $\sum M_{xx}$ and $\sum M_{xx}$ represent the resultant moment of different axes, including righting moment $\mathbf{M}^R = \{M_{xx}^R, M_{yy}^R, M_{zz}^R\}$, damping moment $\mathbf{M}^C = \{M_{xx}^C, M_{yy}^C, M_{zz}^C\}$ and wave exciting moment $\mathbf{M}^w = \{M_{xx}^w, M_{yy}^w, M_{zz}^w\}$; $\sum F_{oz}$ denotes the resultant external force and I means the rotational inertia. The corresponding diagrams of coordinate system are illustrated in Figure 1.

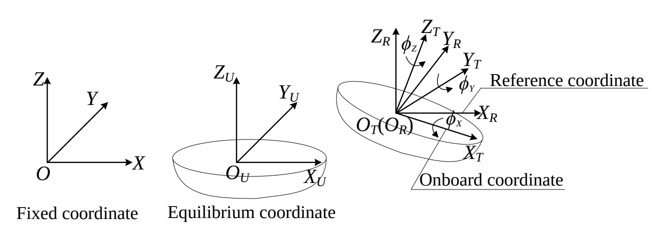

Figure 1: Diagrams of coordinate system for ship.

Assume that the ship's sailing direction is the positive direction of X-axis in the fixed coordinate system, the random oblique wave can be simulated by harmony superposition method, which is

$$f(x,y,t)=\sum_{i=1}^{N} H_i \cos(k_i(x\cos\alpha + y\sin\alpha) - \omega_i t + \beta_i) \tag{20}$$

where

$$\begin{cases} H_i^2 = 2S(\omega)\triangle\omega_i \\ \triangle\omega_i = (\omega_{\max} - \omega_{\min})/(N-1) \\ \omega_i = \omega_{\min} + (i-1)\triangle\omega_i \\ \beta_i = 2\pi\xi_i \end{cases} \tag{21}$$

in which H_i denotes the amplitude, β_i means the initial phase, ξ_i denotes a random variable following uniform distribution within [0, 1]; k is the wave number; ω_i indicates the wave circular frequency; α is the rotation angle of wave; $S(\omega)$ denotes the power spectral density function of wave, simulated by JONSWAP wave spectra (Guachamin-Acero and Portilla-Yandún 2021).

In this example, 14360DWT bulk cargo ship is applied as research object. The main parameters of ship are exhibited in Table 1 and corresponding hull lines of ship are illustrated in Figure 2.

Table 1: Principal dimensions of 14360DWT bulk cargo ship.

Items	*Value*
Length between perpendiculars (m)	*137.0*
Molded breadth (m)	*19.30*
Molded depth (m)	*11.20*
Designed draft (m)	*8.28*
Displacement Δ (t)	*19794.39*

To simulate the random wave, the corresponding parameters are set as: wave length W_l=262m and rotation angle of wave α=30°, wave circular frequency ω_{max}=0.365 rad/s and ω_{min}=0 rad/s. The wave exciting moment $\mathbf{M}^w$ is then obtained by utilizing the surface panel method (Prasad and Dimitriadis 2017).

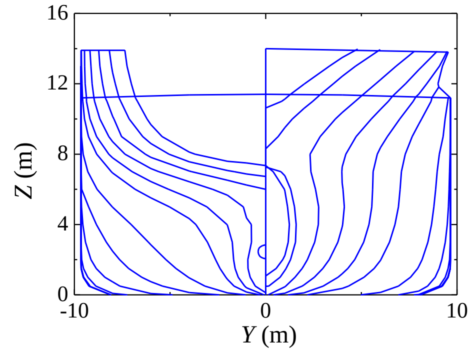

Figure 2: Hull lines of 14360DWT bulk cargo ship.

In the random vibration analysis of MDOF coupled ship model under random oblique wave, the initial conditions of angular velocity and roll angle are set as: $\boldsymbol{\phi}_0$=[0, 0, 0] and $\mathbf{R}_0$=[0, 0, 0]. The stationary PDF curves of ship roll angle R_x under different significant wave heights H_s obtained by DPIM are illustrated in Figure 3. It is shown that the results of DPIM agree well with MCS results, verifying the effectiveness and accuracy of DPIM for nonlinear MDOF system under combined excitation. Moreover, when H_s=2.5 m, the ship roll angle gathers around two periodic attractors, meaning that there exist two possible roll motion states during ship sailing process. However, two periodic attractors are not connected, thus only one of roll motion states is approached when ship starts motion from any initial conditions. And the probability move into right periodic attractor is less than that of left periodic attractor. As H_s increased, the peak of PDF is decreased and the randomness of two attractors is enhanced. Consequently, more possible roll motion states are emerged at H_s=4.0, and the repeated random jump phenomenon between positive and negative roll angle is occurred. It may lead to the loss of ship stability under sail.

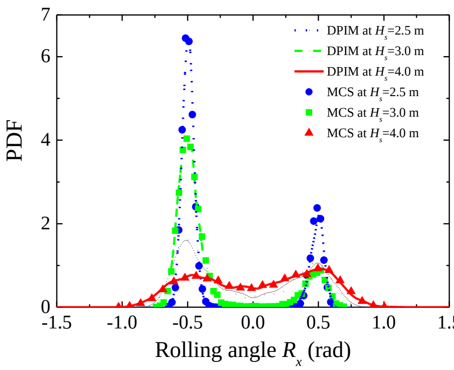

Figure 3: Stationary PDF curves of roll angle R_x *of ship under different significant wave heights* H_s*.*

The safe basin erosion is directly related to the damage of ship, which has been extensively studied in the shipbuilding field (Anastopoulos and Spyrou 2016). Thus, in this example, the global dynamic analysis of MDOF coupled ship model under random oblique wave is further performed by DPIM, and the interest domain is $D=\{(R_x, \phi_x)| -0.5\leq R_x\leq 0.5, -0.5\leq\phi_x\leq 0.5\}$. The angle of vanishing stability is introduced as the safe condition of rolling angle, ensuring $|R_x|\leq\pi/3$ under sail. The 5×5 grid mesh structure is selected for D, where 100 representative points for DPIM and 2500 sample points for MCS are considered in each grid.

Table 2: $\overline{GIM}$*s of safety basins obtained by DPIM and MCS under different significant wave heights* H_s*.*

Methods	*Significant wave height* H_s	$\overline{GIM}$	*CPU time (s)*
DPIM	*2.5*	*0.455*	*5672.59*
	3.0	*0.437*	*6160.63*
	4.0	*0.363*	*7415.78*
MCS	*2.5*	*0.456*	*57944.67*
	3.0	*0.436*	*63243.13*
	4.0	*0.362*	*76389.80*

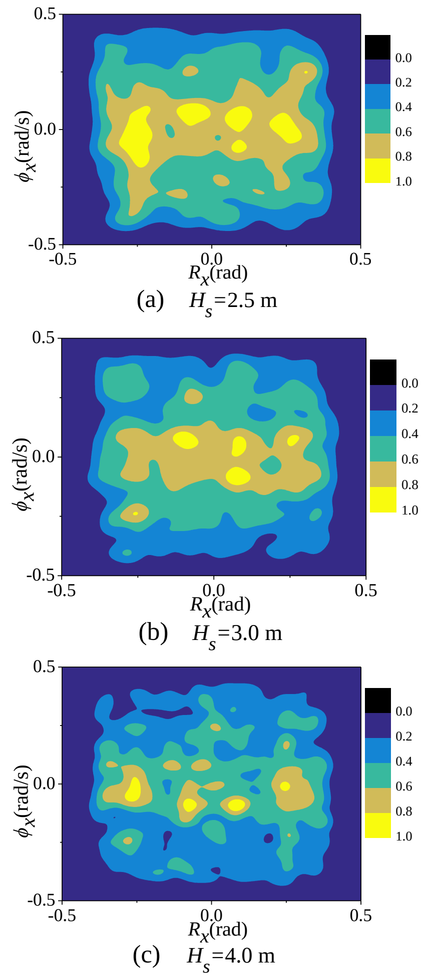

Figure 4: The stochastic safety basins obtained by DPIM under different significant wave heights H_s*.*

The stochastic safety basins of MDOF coupled ship model under different significant wave heights H_s by DPIM are given in Figure 4. Moreover, the corresponding comparisons of $\overline{\text{GIM}}$s of stochastic safety basins obtained by DPIM and MCS, as well as the CPU time of two methods are illustrated in Table 2, demonstrating the high accuracy and efficiency of DPIM. In Figure 4, it can also be observed that the stochastic safety basin at H_s=2.5 m is a continuous area. However, as the increase of H_s, the trajectories in safety region will flow to unsafety region, leading to the stochastic safety basin erosion. This observation implies that the ship's capsizing probability will be increased with significant wave height H_s. The corresponding ship stability loss can be measured by $\overline{\text{GIM}}$ as exhibited in

Table 2, e.g., the $\overline{\text{GIM}}$ of stochastic safety basin at H_s=2.5 m is 0.455 and that at H_s=4.0 m is decreased to 0.363.

5. CONCLUSIONS

In this paper, a novel direct probability integral method is proposed to deal with the random vibration and global dynamic analyses of nonlinear MDOF systems under combined additive and multiplicative excitation.

In the random vibration analysis, by calculating PDIE and governing differential equation of system separately, DPIM obtains response's PDF with high efficiency. Then, for the global dynamic analysis, a new DPIM-based strategy is proposed to obtain the generalized stochastic basin of system and realize its stability assessment, in which the ε-committor function and GIM are reformulated by PDIE of response probability in concerned domain from the viewpoint of probability.

Finally, the random vibration and global dynamic analyses of MDOF coupled ship model under random oblique wave are implemented by DPIM, exhibiting the high accuracy and efficiency of the proposed method. In particular, the ship safety basin is given in a probabilistic manner. Results illustrate that the repeated random jump phenomenon of ship rolling and the stochastic safety basin erosion will be emerged at the higher significant wave height, which may lead the ship to capsize. These observations indicated that the proposed method can be viewed as one of the most powerful approaches to address random vibration and global dynamic analyses of nonlinear MDOF system under the combined additive and multiplicative excitation.

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