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Interpolation of confidence intervals for fatigue design using a surrogate model

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Abstract

For complex systems, the applicability of surrogate models has shown the potential to enable accurate assessments using a reduced batch of data and to compile information about large datasets. These behave as black-box functions that replace a series of inputs/outputs.

In the present work, a Kriging surrogate is used to predict confidence intervals in an offshore wind turbine tower fatigue design. Uncertainty in fatigue due to loading is highly connected to the mean. One year operational fatigue results is used to validate the results. The Kriging is applied to replicate the yearly states of operation, and successfully predicts intervals of confidence for the long-term fatigue design.

Regarding the interest of data analysis, the approach implemented is characterized by its flexibility and capability of approaching any problem that can be characterized by a single variable. Being therefore an interesting tool in decision schemes where large datasets are available or prediction of unknown outputs is required.

1. Introduction

Structural fatigue design is commonly one of the most effort demanding tasks in the whole design process. Because fatigue is a long-term cumulative process of short-term occurrences its assessment during the design phase is limited in probabilistic robustness.

For complex systems, the applicability of surrogate models has shown the potential to enable efficient uncertainty assessments. These can behave as black-box functions that replace computationally expensive codes or data sets. Moreover, surrogate models such as the Kriging models, can accommodate some measure uncertainty by considering Gaussian uncertainty in the interpolated points. Under certain conditions these models can be transformed in full-field stochastic curves that are able to replace complex design models. This is the case of OWTs, where a state of operation may be characterized by multiple output results due to the system's inherent randomness.

Examples of relevant works that use Kriging models to assess a system's performance in a probabilistic way are presented in (Hua et al., 2004; Bichon et al., 2008; Echard et al., 2011; Echard et al., 2013, Yan et al. 2015). In the specific case of OWT Kriging models were also widely applied not only to infer on probabilistic behaviour, but also to reduce computational time (Maki et al., 2012; Morató et al., 2016; Teixeira et al., 2017a).

A methodology that uses a Kriging surrogate model to interpolate fatigue confidence intervals (*CI*) is implemented in the current work. This methodology is of interest for materials for which the

fatigue life does not strongly depend on high stress ranges where statistical extrapolation is compulsory. This is the case of the steel components, widely applied in the design of OWT towers.

For the proposed goal Section 2 introduces the theory behind the Kriging surrogate models, Section 3 discusses fatigue life design, Section 4 presents the results for the application of Kriging models as interpolators of the CI , and finally, Section 5 presents the main conclusions of the work underpinned.

2. Kriging surrogate models

Kriging models can be implemented as “black-box” models that interpolate a data set with the assumption that the interpolator follows a Gaussian stochastic process.

These approximate a true function $g(x)$, $x \in \mathbb{R}^d$ with an approximate model $G(x)$ that considers Gaussian uncertainty in the approximation. d is the dimension of the x space.

Being $\mathbf{X} = [x_1, x_2, \dots, x_k]$ a vector of realisations of x with respective true evaluations $\mathbf{Y} = [g(x_1), g(x_2), \dots, g(x_k)]$, $G(x)$, the Kriging approximation, is defined as,

$$G(x) = f(\boldsymbol{\beta}; x) + Z(x)$$

$$f(\boldsymbol{\beta}; x) = \beta_1 f_1(x) + \dots + \beta_p f_p(x)$$

$f(\boldsymbol{\beta}; x)$ is a deterministic regression model with p ($p \in \mathbb{N}^+$) basis function that depend on the order of the regression and that are to be defined based on the \mathbf{X} sample.

$Z(x)$ is a Gaussian stochastic process with mean equal to 0 and covariance matrix,

$$\mathbf{C}(x_i, x_j) = \sigma^2 R(x_i, x_j; \boldsymbol{\theta}), \text{ with } i, j = 1, 2, 3 \dots k$$

\mathbf{C} defines the correlation generic \mathbf{X} points using; the constant process variance σ^2 and a correlation function $R(x_i, x_j; \boldsymbol{\theta})$ that depends on the points and on a set of $\boldsymbol{\theta}$ hyperparameters. It is common to find application that use the so-called *separable* correlation function (Roustant et al. 2012),

$$R(x_i, x_j; \boldsymbol{\theta}) = \prod_{i=1}^d R(h_i; \theta_i), \boldsymbol{\theta} \in \mathbb{R}^d$$

Alternative formulations for the correlation function can be found in (Rasmussen, 2004; Marelli and Sudret, 2014). $R(h_i; \theta_i)$ depends on $h = [h_1, \dots, h_d]$ that has an incremental form and a set of $\boldsymbol{\theta}$ hyperparameters. $R(x_i, x_j; \boldsymbol{\theta})$ interpolates the between the regression model prediction and the true limit state realisations.

Using the equations above it can be seen that the fitted surface $G(x)$ depends on three main parameters: σ^2 , $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$.

For a set of given support points \mathbf{Y} the problem of prediction can be solved through a generalised least squares formulation. $\boldsymbol{\beta}$ and σ^2 are then defined as a function of the $\boldsymbol{\theta}$ hyperparameters,

$$\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}(\boldsymbol{\theta}) = (\mathbf{F}^T \mathbf{C}^{-1} \mathbf{F})^{-1} \mathbf{F}^T \mathbf{C}^{-1} \mathbf{Y}$$

$$\sigma^2 = \hat{\sigma}^2 = \frac{1}{k} (\mathbf{Y} - \mathbf{F} \hat{\boldsymbol{\beta}}(\boldsymbol{\theta}))^T \mathbf{C}^{-1} (\mathbf{Y} - \mathbf{F} \hat{\boldsymbol{\beta}}(\boldsymbol{\theta}))$$

Where \mathbf{F} is the $k \times p$ regression matrix that has rows equal to the evaluation of the p regression functions at \mathbf{X} . The problem of finding $\boldsymbol{\theta}$ can be solved by a maximum likelihood formulation.

As a result a prediction for $g(x)$ in a generic point in space u is given by an expected value μ_G and a variance σ_G ,

$$\mu_G(u) = \mathbf{f}^T(u) \boldsymbol{\beta} + \mathbf{c}^T(u) \mathbf{C}^{-1}(\mathbf{Y} - \mathbf{F}\boldsymbol{\beta})$$

$$\sigma_G^2(u) = \sigma^2 \left(1 + D^T(u)(\mathbf{F}^T \mathbf{C}^{-1} \mathbf{F})^{-1} D(u) - \mathbf{c}^T(u) \mathbf{C}^{-1} \mathbf{c}(u) \right)$$

$$D(u) = \mathbf{F}^T \mathbf{C}^{-1} \mathbf{c}(u) - \mathbf{f}(u)$$

$\mathbf{c}(u) = c(u, x_i), i = 1, 2, \dots, k$ is a vector that defines the correlation of u with the known points of \mathbf{X} . \mathbf{F} is the matrix of $f_p(x)$ functions of the polynomial approximation.

One important characteristic of the Kriging interpolator is that \mathbf{Y} predictions have a variance component of value 0, meaning the \mathbf{Y} is exactly predicted when u takes any value of \mathbf{X} .

It is known that in most cases this does not represent the real behaviour of \mathbf{Y} and frequently $g(\mathbf{X})$ will have some noisy or probabilistic component ε .

$$\mathbf{Y} = \mathbf{Y}_e + \varepsilon$$

In such cases \mathbf{Y} is better described by an expected value \mathbf{Y}_e and a noise component ε . To account for this effect a slight modification of the $\mathbf{C}(x_i, x_j)$ is introduced,

$$\mathbf{C}(x_i, x_j) = \mathbf{C}(x_i, x_j) + \boldsymbol{\delta} \tau^2$$

where τ^2 is the vector of variance of \mathbf{Y} , the support points used to estimate $G(x)$. $\boldsymbol{\delta}$ is the identity matrix of size $k \times k$. An example of both the non-noisy and noisy approach to \mathbf{Y} are presented in

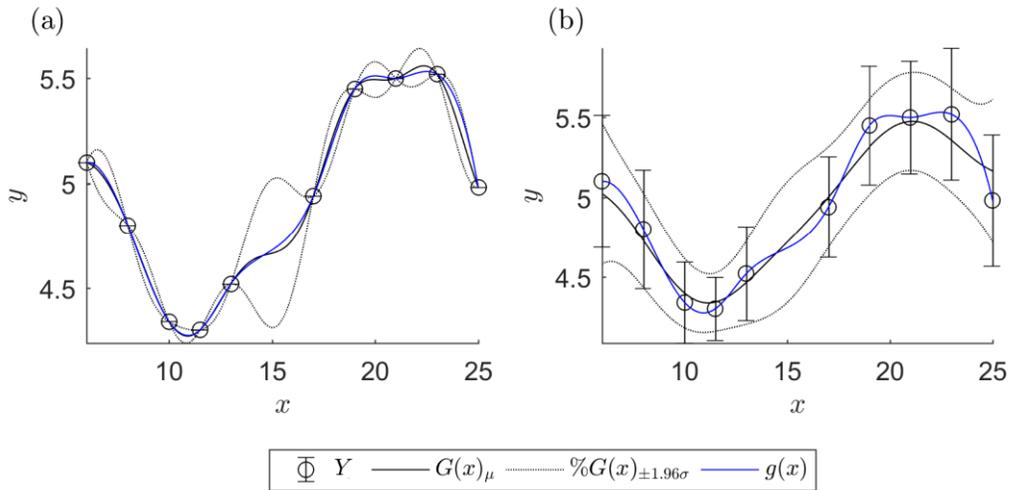


Figure 1 – Example of Kriging considering non-noisy \mathbf{Y} (a) and noisy \mathbf{Y} (b) for the same \mathbf{X} .

If the goal is to accurately approach $g(x)$ the usage of a noisy scheme may be questioned, however there are numerous applications where a noisy Kriging is of interest.

To utility of the noisy Kriging to replicate a probabilistic field is exploited in the current paper.

3. Fatigue analysis of OWT towers

The recommended calculation of fatigue for OWT involves linearly summing the contribution to the lifetime (T) damage (D_T) of different load ranges (S) with Miner's rule (IEC 2005, 2009). D_T is, in a straightforward approach, commonly calculated using

$$D_T = \sum_{i=1}^{L_n} \frac{n_E(S_i)}{n_{SN}(S_i)}$$

where the calculation is based on counting the expected number of cycles, $n_E(S_i)$, at a certain stress range and comparing it with the allowable number of cycles $n_{SN}(S_i)$ accordingly to a specified SN curve.

The assessment of $n_E(S_i)$ frequently involves using a cycle counting technique, e.g. rainflow counting, with a limited number of simulations. The results are then scaled up for T .

In ideal circumstances, in the estimation of fatigue design life, data corresponding to T operation points should be analysed. However, this is not practical and fatigue life is calculated using a limited number of data within different a set of Θ operation conditions. To assess D_T coming from the different Θ states an integration procedure takes place,

$$D_T = \int_{\Theta} \int_S \frac{f(\Theta)f(S|\Theta)}{n_{SN}(S)} dS d\Theta$$

where the distribution function of operational conditions $f(\Theta)$ is used with the conditional distribution of stress ranges $f(S|\Theta)$. This distribution is estimated from the stresses and cycles obtained at Θ , being conditional on it.

As multiple repetitions occur within T ; fatigue in OWT towers can be treated as a problem of mean (Lange, 1996). $G(x)$ is applied to take advantage of this fact and interpolate uncertainty using a limited number of Θ and then, with $f(\Theta)$, estimating D_T considering its uncertainty.

If information about a complex process can be comprised in a single indicator, it can therefore be approximated with a Kriging surrogate model. In addition, this same model can be used to represent its uncertainty. This is the idea behind characterizing uncertainty in T using damage values calculated at a t time, shorter than T (D_t).

The idea of using the Kriging model is thus, interpolating the confidence intervals over all the $x \subset \Theta$ domain, and as a result, define the long term confidence intervals for D_T .

The analysis is developed using NREL 5MW baseline monopile OWT (Jonkman et al. 2009). A double slope SN curve given by (DNV, 2014) is used as representative of the tower material.

IEC (2005, 2009) recommends the usage of 6 seeds to assess the Design Load Case related to operational fatigue. Further discussion on the representability of the sample to estimate D_T is given in Sutherland (1999). In the current work 10 seeds are implemented.

Regarding the statistical characterization of the t damage, Teixeira et al. (2017b) showed that D_t in an OWT follows a lognormal distribution. This assumption is of relevance to estimate confidence

intervals in the mean using a Kriging model. Therefore, onwards, μ_{LN} and σ_{LN} define the first two lognormal moments of D_t , whereas μ_N and σ_N is used to refer to the first two statistical moments of the respective lognormal's associated normal distribution.

4. Confidence interval interpolation.

To estimate CI , as stated 10 simulations with distinct seeds are performed for each Θ . These are then used to calculate the local CI s of μ_N .

The initial sample size is generated selecting subset points of a Latin Hypercube Sampling scheme in $f(\Theta)$, with addition of extreme occurrences of Θ . Θ is defined as function of the wind velocity (U) and turbulence intensity (I). Teixeira et al. (2017b) showed these to be the most influential variables in the baseline turbine tower design.

The initial sample and respective margins with the confidence intervals of the mean are presented in Table 1. \mathbf{X}_{i+1} refers to new points that are added to the initial \mathbf{X} , Figure 2.

The confidence intervals in every iteration of \mathbf{X} are calculated using a t-student distribution with $N-I$ degrees of freedom (DF), being N the number of points used to estimate μ_N , the mean of the associated Normal distribution. The critical value for the 95% confidence intervals of a t-distribution with 9 Degrees of Freedom (DF) is 2.262. The limits of confidence (CI_{95}) for the mean value of D are given by:

$$CI_{95} = \mu_D \pm t_{95} (DF = N - 1) \frac{\sigma_N}{\sqrt{N}}, \quad \text{with } t_{95} (DF = 9) = 2.262$$

Table 1 – Initial DoE \mathbf{X} used in both the analysed cases (a) and (b) of CI interpolation. $\pm\Delta CI$ refers to the variation of the confidence interval that is given by the 2nd term of the right side of the CI calculation.

Initial \mathbf{X} of case (a)				Initial \mathbf{X} of case (b)			
U(m/s)	I(%)	$\mu_N(g(x = \Theta))$	$\pm\Delta CI_{95g(x)}$	U(m/s)	I(%)	$\mu_N(g(x = \Theta))$	$\pm\Delta CI_{95g(x)}$
5.43	2.15	-24.813	0.193	5.43	2.15	-24.813	0.193
5.43	7.12	-23.200	0.216	5.43	7.12	-23.200	0.216
5.43	10.70	-21.884	0.190	5.43	10.70	-21.884	0.190
5.43	5.65	-21.329	0.425	7.70	9.15	-20.022	0.534
10.05	5.65	-19.629	0.597	11.45	3.25	-17.641	0.484
12.05	7.37	-14.766	0.170	11.95	4.61	-15.631	0.231
13.98	2.15	-22.057	0.277	12.05	7.37	-14.766	0.170
17.68	2.47	-22.184	0.179	17.68	2.47	-22.184	0.179
18.61	4.27	-20.011	0.139	21.76	4.42	-19.137	0.110
24.86	4.27	-18.751	0.139	24.86	10.70	-14.081	0.205
24.86	10.70	-14.312	0.174	24.86	4.27	-18.751	0.139

\mathbf{X} and its respective iterations for each case were defined using the active learning technique. The selection of new points is supported by an infill criteria that combines, the Expected Improvement introduced by Jones et al. (1998), a spatial, and probability density component that uses the joint distribution of Θ , $f(\Theta)$.

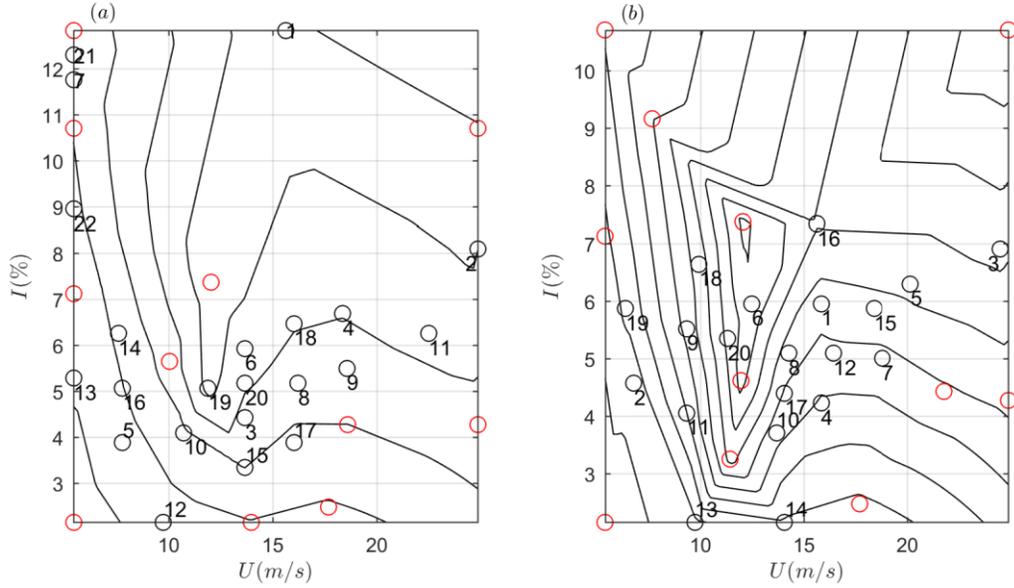


Figure 2 – Initial Design of Experiments X used in the calculation of long term CI of fatigue cumulative damage and active learning iterations X_{i+1} .

The extreme occurrences are used to enclose the space of X . (b) represents an extension of the x in the region of low I , that is more prominent in offshore climates. Few operational points occur outside of the domain of (a) and (b). When this occurs these are extrapolated from the limit region of the Kriging using the nearest point.

A cross validation was used to compare the accuracy of the $G(x)$ in approaching $g(x)$, which in the current case represents D_t . 15 points were picked from x to do the cross-validation exercise. The results are presented in Table 2.

Table 2 – Cross-validation results of the CI interpolation using the final X of the (b) Kriging design of Figure 2. $\pm\Delta CI$ refers to the variation of the confidence interval that is given by the 2nd term of the right side of the CI calculation.

i	θ_i		$\mu_{N_{G(x=\theta_i)}}$	$\mu_{N_{g(x=\theta_i)}}$	$\pm\Delta CI_{95_{G(x)}}$	$\pm\Delta CI_{95_{g(x)}}$
1:	$U = 5.4\text{m/s}$	$I = 6.2\%$	-23.420	-23.543	0.241	0.225
2:	$U = 7.4 \text{ m/s}$	$I = 8.8\%$	-20.396	-20.491	0.774	0.437
3:	$U = 8.7 \text{ m/s}$	$I = 6.5\%$	-20.034	-19.925	0.746	0.451
4:	$U = 9.9 \text{ m/s}$	$I = 7.2\%$	-18.451	-18.243	0.512	0.330
5:	$U = 10.7 \text{ m/s}$	$I = 9.2\%$	-15.865	-16.270	0.804	0.416
6:	$U = 11.3 \text{ m/s}$	$I = 5.6\%$	-16.189	-16.404	0.340	0.302
7:	$U = 12.1 \text{ m/s}$	$I = 7.6\%$	-14.589	-14.568	0.267	0.144
8:	$U = 13.7 \text{ m/s}$	$I = 8.3\%$	-15.010	-15.083	0.245	0.287
9:	$U = 15.3 \text{ m/s}$	$I = 5.8\%$	-18.281	-18.294	0.180	0.245
10:	$U = 16.1 \text{ m/s}$	$I = 6.4\%$	-17.929	-18.019	0.292	0.230
11:	$U = 17.1 \text{ m/s}$	$I = 4.5\%$	-19.751	-19.757	0.189	0.190
12:	$U = 18.3 \text{ m/s}$	$I = 6.0\%$	-18.488	-18.354	0.152	0.169
13:	$U = 19.1 \text{ m/s}$	$I = 5.4\%$	-18.733	-18.728	0.108	0.153
14:	$U = 19.8 \text{ m/s}$	$I = 6.8\%$	-17.263	-17.418	0.403	0.195
15:	$U = 22.4 \text{ m/s}$	$I = 4.8\%$	-18.936	-18.702	0.501	0.160

For all the points of cross validation $\mu_{N_{G(x=\theta_i)}}$ and $CI_{95_{G(x)}}$ predicts the interval that encloses the $\mu_{N_{g(x=\theta_i)}}$. Similarly, $\mu_{N_{g(x=\theta_i)}}$ and $CI_{95_{g(x)}}$ also encloses the interpolated $\mu_{N_{G(x=\theta_i)}}$. One of the

particularities of the Kriging is the increase of CI prediction for the points where less is known about \mathbf{X} . Examples of this can be found for Θ_5 , Θ_{14} and Θ_{15} , where the error in the interpolated μ_N is higher and the CI is also over-predicted. In some case, such as Θ_2 and Θ_3 , the approximation to $\mu_{N_{g(x=\theta_i)}}$ is more accurate but the confidence intervals are over-estimated.

As the infill technique considers $f(\Theta)$ it focuses on areas of Θ that are of relevance for the operation of the OWT. Less confidence is expected in such regions as less points of x will be analysed in these areas. This approach improves the computational efficiency of the method, and is based in the assumption that fatigue is a problem of repetition and some Θ states will have limited effect on the T fatigue design uncertainty. This is the case of the points Θ_4 and Θ_5 , that, due to the offshore wind less turbulent characteristics are less likely to happen. When performing cross-validation to infer and improve on the model accuracy a critical analysis on the contribution of each point to D_T is demanded. The trade-off in improving the model in some region of x may not be positive. The uncertainty in the confidence intervals can also be used to drive new iterations of \mathbf{X} .

If more accuracy is demanded it can be attained by calibrating the θ parameters, Figure 3.

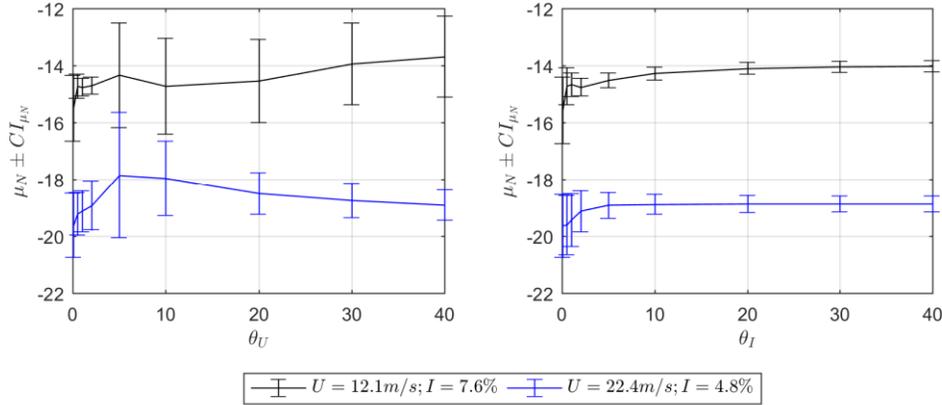


Figure 3 –Influence of $\theta_{i=1,\dots,d}$ on the interpolated CI for two of the cross-validation points considered, when maintaining a constant interval for $\theta_{j \neq i}$, with $j \in i = 1, \dots, d$, of $[0.5, 2]$.

By analysing the Kriging model formulation it can be seen that these two parameters relate to U and I such a way that they weight the prediction and correlation between points. By understanding the sensitivity of the model to these two variables, CI can be imposed to control the uncertainty in the points where less is known. For the case of I , there is a monotonic relation between the increase in CI and the increase of θ_2 ($= \theta_I$). This relationship can be exploited in order to use θ_1 ($= \theta_U$) to exploit improvements in the interpolation of CI , particularly in the cases where CI are over-estimated or under-estimated.

The calibration of θ occurs simultaneously for all the surrogate, as a result, the approximation of CI can be quite case specific.

The results for the interpolated CI are shown in Figure 4. 2nd order polynomial function were used to build the Kriging. The convergence limits for θ were set in between 0.5 and 2. These are compared with the damage generated by a one-year full assessment consisting of 51240 simulations $t = 10$ minutes (D_T^r) based on environmental data presented on Teixeira et al. (2018a). The lognormal associated long term normal mean is used in order to have symmetric representation of the CI .

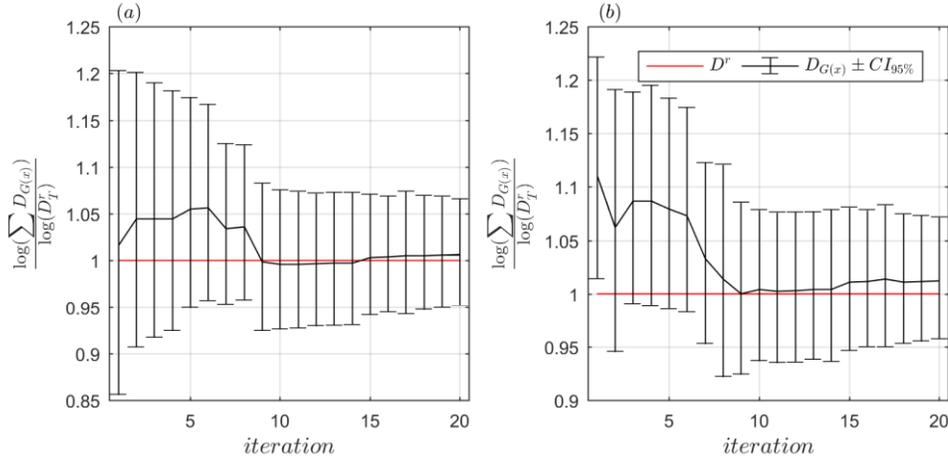


Figure 4 – Interpolated confidence interval for the long term damage and comparison with the real value calculated from a one-year simulation. The results are respective to Figure 2 (a) and (b) X.

Even when a small number of iterations is conducted to improve the approximation to $g(x)$, D_r is within the estimated CI. As the number of iterations increases the CI decreases. The interest of improving the surrogate is mainly connected to a better estimation of the CI.

The results obtained for (a) and (b) are expected to be on the safe side of the design procedure. It is unlikely, when calculating a mean in multiple points, for the deviation of the estimated mean in relation to the true mean to be always negative and positive and have always the value of the CI bounds. This deviation, when $k \rightarrow \infty \in \mathbf{X}$, is expected to average over 0. Nevertheless, it is common for k to be limited by practical reasons. Furthermore, different Θ have different weights in contributing to D_T . Therefore, if one or more very damaging Θ are under-estimated in their contribution to D_T , erroneous lifetime designs can be obtained.

A more reasonable approach would be to generate a high number of surfaces from the Kriging model and then infer on the variability of these surfaces in estimating D_T . Each sampled surface represents a full design process considering that the mean in each point will always be within its 95% intervals. Estimating D_T using the damage of different surfaces (D_{surf}), Figure 5, replicates the uncertainty originated by the design process where each surface represents a single calculation of D_T depending on different mean values that can result from using 10 seeds to calculate $f(S|\Theta)$ within $f(\Theta)$. In the limit this variability will be comprised in between the CIs bounds of Figure 4.

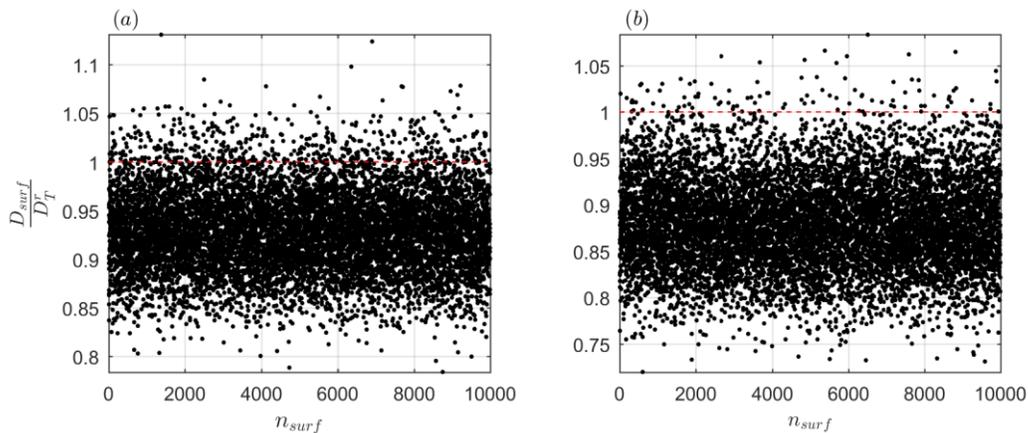


Figure 5 – Results for 100000 design surfaces sampled with the Kriging results from case (a) and (b) final iterations.

In both cases (a) and (b) the multiple surfaces given by the *CI*s are able to enclose D_T^r . Nevertheless, in (b) the final iterated Kriging mean was less accurate in predicting D_T^r . When the $G(x)$ prediction is not accurate the *CI* and variability of the surfaces of design damage are higher. Sampling surfaces or using *CI*s can be used to impose limits of D_T^r . To note that *CI* estimated with the bounded values may lead to conservative design, but are guaranteed all the occurrences within the *CI*.

The methodology presented in the current paper is of interest due to its simplicity and flexibility. A large amount of data can be contained in a Kriging surrogate. It can help to predict x points where little is known about the real performance function. Furthermore, these models can be implemented in any circumstance where the information about the problem can be compiled in a representative indicator. Further implementations should consider the expansion of the methodology in order to accommodate important uncertainty sources such as uncertainty in the SN curve definition.

5. Conclusions

The present paper evaluated the usage of Kriging surrogate models as an approximation of confidence intervals for long term structural fatigue design assessment. Structural fatigue design is a problem of mean value. The fatigue design value converges statistically and the variation of the long term fatigue depends mainly on the accuracy of the mean estimation. The Kriging can then be used as a surrogate that approximates the mean value, and interpolates the uncertainty in its estimation for all the points in the space of variables that influences the system's long-term fatigue.

Results from the surrogate approximation were compared with a full one year simulation. Two case study enclosing 310 simulations were analysed. The interpolated confidence intervals enclosed the real value of a full one year damage operation in the both cases.

The methodology used is of interest to have a measure uncertainty in the estimation of the long term fatigue design damage. The long-term design damage is highly dependent on the x points used to set the load distributions, and as a result, deviations in the estimation are also highly related to x .

By using the confidence intervals and the surrogate model, not only uncertainty is quantified, but also significant design time can be cut from the design procedure.

To conclude, it is important to highlight that the methodology implemented is characterized by its flexibility and potential of application in other fields of knowledge, being the minimum requirement for it the capability of defining what is described as a relevant indicator that can be interpolated by the Kriging. It can then be used as an effective tool to present in a succinct basis information about a specified process and help decision making schemes with awareness of the different uncertainties that subjacent it.

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