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Atterberg limits and remolded shear strength—water content relationships

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Abstract:

The remolded undrained shear strength (s_{ur}) is of importance in many geotechnical applications. This paper uses published regression analyses for strength variation with water content (w) to draw some generalized conclusions on soil strength behavior, including values of s_{ur} mobilized for the Casagrande liquid limit (LL) and plastic limit (PL) conditions and hence the shear strength variation occurring over the plastic range. Reported methods of deducing strengths from measured values of water content (w) at the Atterberg limits are reviewed, many of which assume a strength variation of 100 over the plastic range. Regression analysis of reported $\log w$: $\log s_{ur}$ correlations for 14 mineral soils and four organic sediments indicated that the strength variation is generally not 100 and can vary significantly between soils. Hence a new approach is proposed for predicting values of remolded undrained strength mobilized for different water contents. Whereas existing formulations are based on empirical s_{ur} values associated with the Atterberg limits, the proposed approach is based on direct strength measurements along with water contents for two test-specimens.

Keywords: Atterberg limits; correlation; clay; organic; plasticity; strength

1. INTRODUCTION

The remolded undrained shear strength (s_{ur}) is of importance in many geotechnical applications including pile design, studies of glacial soils and submarine soil investigations for offshore structures (Kayabali and Tufenkci, 2010). Remolded soils are widely used in the construction of earthworks. In addition, geotechnical engineers often have to make important design decisions based upon inadequate and (or) poor quality soil strength data. In such cases, the engineer may wish to evaluate s_{ur} as a lower bound (conservative) value for the undisturbed strength of the soil deposit. Correlations can provide indicative values against which data from more sophisticated strength tests performed on undisturbed specimens can be assessed with an inbuilt safety margin (Wood 1990, Sharma and Bora 2003) and can also allow inference regarding possible levels of specimen disturbance.

This paper considers the field of proposals and interpretations for deducing s_{ur} values of plastic saturated clay-rich soils based on measured values of water content (w), liquid limit (LL) and plastic limit (PL). Values of LL and PL are usually determined using the Casagrande percussion-cup and thread rolling methods, respectively (Casagrande 1932, 1958), although fall-cone approaches are preferred in some countries. This paper has three main objectives:

- Commonly used methods for predicting values of s_{ur} for different water contents within the plastic range on the basis of measured LL and PL values are critically assessed.
- A review of shear strength values mobilized at the Casagrande LL and PL conditions and hence the strength variation over the plastic range;

- A new approach is proposed for the prediction of s_{ur} values mobilized for different water contents within the plastic range.

Many empirical relationships between water content and s_{ur} have been reported in literature (e.g. see Yang et al. (2006) and Eqs 1a–1j in Table 1). Water contents are often related to the Atterberg limits in terms of liquidity index (I_L), thereby giving an indication of soil consistency, with I_L values of zero and unity corresponding to PL and LL conditions respectively.

Table 1. Selection of published strength–water content correlations for remolded soils. *Note: Table 1 contains Equations 1(a) to 1(j).*

However it has been well documented that the I_L against $\log s_{ur}$ relationship is non-linear (Skempton and Northey, 1953, Wasti and Bezirci, 1985, Koumoto and Houlsby, 2001); exhibiting a concave curvature over the full plastic range (see Fig. 1), with data for different soils also plotting along different curves. Wood (1990) has shown that critical-state theory provides a rational basis for many of these correlations, in particular Eq. (1a), although this equation assumes that the spread and curvature of the I_L against $\log s_{ur}$ data can be neglected.

Figure 1. Liquidity index against remolded undrained strength (adapted from Koumoto and Houlsby, 2001).

From theoretical analysis of the cone penetration linking penetration resistance to strength in the fall cone test, and building upon the relationship after Butterfield (1979), Koumoto and Houlsby (2001) proposed the empirical result given by:

$$w = a s_{ur}^{-b} \quad (2)$$

where coefficient a (in %) is the water content corresponding to $s_{ur} = 1$ kPa and coefficient b is the gradient of the water content against strength correlation presented on a bi-logarithmic plot (see Fig. 2).

Figure 2. Water content (in %) against remolded undrained strength (in kPa) for saturated soil.

The coefficient a in Eq. 2 relates to the soil's water absorption/water-holding capacity and hence depends mainly on its grading and the mineralogy, shape, surface texture and activity of the clay minerals. The coefficient b relates to the soil compressibility. This $\log w$: $\log s_{ur}$ relationship has also been investigated experimentally and reported valid over the full plastic range for both amorphous organic (Zentar et al., 2009, O'Kelly, 2013a) and fine-grained mineral soils. In particular, Sharma and Bora (2003) reported that this relationship, as determined experimentally for 55 inorganic soils having LL values in the range 33–82%, was linear over the full plastic range and that this relationship also extended for $w > LL$. A reported exception was montmorillonite soils (tested for $LL = 210$ – 460%), which exhibited a bi-linear relationship at values of water content near LL. Like Atterberg limits, the values of the soil-dependent coefficients a and b are strongly dependent on the size and quantity of clay minerals in the soil composition and also the quantity of interlayer water in the case of expanding clay minerals (Trauner et al. 2005). The values of these coefficients are usually determined from regression analysis of strength data measured over a range of water contents, but they can be calculated from values of the critical-state strength and compressibility parameters (e.g., see Koumoto and Houlsby (2001)). Trauner et al. (2005) showed that the values of these coefficients can be also determined from knowledge of mineralogical properties.

Referring to Fig. 2, values of s_{ur} can be interpolated for water contents within the plastic range (and possibly also extrapolate strengths beyond this range) using Eq. 2. Also, the empirical expressions given by Eqs. 3 and 4, which are equivalent to Eq. 2, can be used to interpolate values of s_{ur} from measured water contents corresponding to the LL and PL conditions, provided the respective strengths of $s_{ur(LL)}$ and $s_{ur(PL)}$ can be correlated with one another, either in terms of the value of coefficient b or the strength ratio R_s (i.e., $s_{ur(PL)}/s_{ur(LL)}$) occurring over the plastic range.

$$\log s_{ur} = \log s_{ur(LL)} + \frac{1}{b} \log \left(\frac{LL}{w} \right) \quad (3)$$

$$\log s_{ur} = \log s_{ur(LL)} + \frac{\log R_s}{\log(LL/PL)} \log \left(\frac{LL}{w} \right) \quad (4)$$

Referring to Fig. 2, the strength ratio R_s and coefficient b can be related by

$$R_s = \exp \left(\frac{I_{PN}}{b} \right) \quad (5)$$

where the modified plasticity index, $I_{PN} = \log LL - \log PL$.

Given the bi-logarithmic strength against water content relationship, it was logical to define a modified liquidity index (I_{LN} , Eq. 6) that produces a linear relationship against logarithm of s_{ur} (Fig. 3). Also as evident from closer inspection of Fig. 3, although the experimental data for a given soil are linear, they are not coincident since different soils generally have different R_s values (see later).

$$I_{LN} = \frac{\log w - \log PL}{\log LL - \log PL} \quad (6)$$

Figure 3. Modified liquidity index against remolded undrained strength (adapted from Koumoto and Houlsby (2001)).

Some researchers (e.g. Stone and Phan (1995), Feng (2000, 2001), Sharma and Bora (2003)) have reported mean values of $s_{ur(LL)} \approx 1.7$ kPa and $s_{ur(PL)} \approx 170$ kPa for many mineral soils and concluded that $R_{1.7} \approx 100$; where $R_{1.7}$ is the strength ratio determined on the basis of $s_{ur(LL)} = 1.7$ kPa. The value of $R_s \approx 100$ appears to have been attributed to Skempton and Northey (1953) in the first instance, although these authors did not explicitly report or infer this concept in their paper. However, closer examination of their experimental liquidity index against strength data plots indicates that the LL and PL conditions did not correspond to these strength values and also that respective R_s values for the Horten, London and Shellhaven clays (see Table 2) considered were 88, 120 and 128. It is important to emphasize that there is no theoretical basis for a value of $R_s \approx 100$ between the Casagrande LL and PL conditions. The strength of soils at their Casagrande LL varies with LL on account of the associated change in density (Youssef et al., 1965; Haigh, 2012a). Nagaraj et al. (2012) and Haigh et al. (2013) reported that no unique value of undrained strength can be assigned at the Casagrande PL. Based on a review of available data for 71 soil samples from published literature, Haigh et al. (2013) concluded that $s_{ur(PL)}$ could vary between 17 and 530 kPa. Barnes (2009), for instance, reported that Casagrande PL establishes the limit of soil workability and is therefore dependent on soil toughness, rather than a notional strength value of ~ 170 kPa. Experimental data have been reported in literature to support both viewpoints. For example, clay and fine quartz-sand mixtures, with the clay fraction comprising between 25% and 100% of either kaolinite or montmorillonite by dry mass, had reported values of $R_{1.7}$ nearer 30 and of the order of 100 respectively (Dumbleton and West, 1970) from which Wood (1990) concluded that the value of R_s is probably dependent on the activity of the clay mineral present. Whyte (1982) reported that for some Swedish clays, $s_{ur(LL)} = 1.6$ kPa, $s_{ur(PL)} = 110$ kPa and hence $R_s \approx 70$, whereas Karlsson (1977) reported $R_s \approx 50-100$. Sivakumar et al. (2009) reported that the upper end for the range of R_s values was 170. Some researchers who support the position of $R_s \approx 100$ have attributed the wide range in reported R_s values mainly to the fact

that measurement of the value of $s_{ur(LL)}$ is not straightforward (Sivakumar et al., 2011), data are inevitably scattered and the disparity of measurement approaches used have together given rise to the relatively wide $s_{ur(LL)}$ range reported.

One of the aims of the present study is to investigate the strength values mobilized at the Casagrande LL and PL conditions and hence the strength ratio R_s considering a range of different soils. As already mentioned, measurement of the very low strength values mobilized for water contents about the LL condition is problematic, with reported values invariably determined using miniature vane tests, although the interpretation of such data is open to question (Koumoto and Houlsby, 2001, Kayabali and Tufenkci, 2010). Whether the vane failure mechanism is the same in soil at its LL and PL conditions is open for debate. Unconfined triaxial compression tests are not possible at water contents around the LL condition. Also, specimen preparation is tricky around the PL condition because of the soil's generally stiff consistency and it is also difficult to prepare saturated specimens. Unsaturated specimens reconstituted at PL would mobilize strengths greater than $s_{ur(PL)}$.

Hence rather than measuring $s_{ur(LL)}$ and $s_{ur(PL)}$ values directly for a range of different soils, it was decided to adopt a different approach in the present study. Following a review of the literature (see Table 2), respective vane shear strengths corresponding to the reported Casagrande LL and PL values were determined from published coefficient a and b values for different soils using Eq. 2, along with values of the strength ratio R_s . Note that since the analysis works on the regression parameters, rather than raw data, it assumes the shape of the fit function, in this case a bi-logarithmic strength variation with water content.

The data set was comprised of 14 mineral fine-grained soils and four dredged sediments having an OC = 6.7–9.7%; where OC is the organic content. Also, two of the mineral soils considered were bentonite, with reported Casagrande LL values of 334% and 402%.

Table 2. Database of Atterberg limits and strength coefficients a and b .

2. PROPOSED WATER CONTENT–UNDRAINED STRENGTH RELATIONSHIP

Many existing formulations (e.g. Eqs. 1a, 1g, 3 and 4) are based on empirical s_{ur} values associated with the LL and PL conditions which are correlated with one another in terms of an assumed strength ratio R_s value. A value of $R_{1.7} = 100$ (Stone and Phan 1995, Feng 2000, 2001, Sharma and Bora 2003) is often adopted, although the value of R_s appears to vary widely between different soils (e.g., see Haigh et al. (2013)). Hence the reliability of s_{ur} predictions derived using Eq. 4 for $s_{ur(LL)} = 1.7$ kPa and $R_s = 100$ is questionable, even in obtaining first approximation values. Furthermore, referring to Fig. 3, although linear, the experimental $I_{LN} - \log s_{ur}$ data plots are not coincident on account of different soils generally

having different experimental R_s values. Hence a new approach is proposed for the prediction of remolded undrained shear strength at different water contents within the plastic range, building upon the $I_{LN} - \log s_{ur}$ relationship encapsulated by Eqs. 1(g) and 6, the former reported in Table 1.

The proposed approach is equally applicable for both fine-grained mineral and amorphous organic soils, for which the $\log w : \log s_{ur}$ relationship has been reported valid. However the proposed method does not link the required strength value corresponding to a specified water content in terms of the strengths mobilized at the Atterberg limits. Control data are obtained for the soil under examination from direct strength measurements (e.g. shear vane or unconfined triaxial compression tests) on two test-specimens, *A* and *B*. These specimens are prepared at arbitrary water contents, but ideally close to either end of the plastic range. The wider the range of strengths associated with specimens *A* and *B*, the more reliable the interpolation of strengths for intermediate water contents. From these strength and corresponding water content measurements, values of s_{ur} for the soil can be deduced for any value of water content within the plastic range and vice versa. From Eq. 4, it is clear that one should work with ratios of water contents rather than differences. Hence, analogous to the modified liquidity index (Eq. 6), new parameters of relative water content W_{LN} and relative strength $I_{s_{ur}}$ are introduced and defined as

$$W_{LN} = \frac{\log w - \log w_A}{\log w_B - \log w_A} \quad (7)$$

$$I_{s_{ur}} = \frac{\log s_{ur} - \log s_{urB}}{\log s_{urA} - \log s_{urB}} \quad (8)$$

where s_{urA} , s_{urB} are measured undrained strengths and w_A , w_B are corresponding water contents of test-specimens *A* and *B*, respectively, with $w_A < w_B$

Note the values of W_{LN} and $I_{s_{ur}}$ have purely relative meaning and are not tied to any standard test or procedure. In this manner, the measured data pairs of (s_{urA}, w_A) and (s_{urB}, w_B) are transposed to coordinates (1, 0) and (0, 1), respectively, on a normal W_{LN} against $I_{s_{ur}}$ plot.

Given that the $\log w : \log s_{ur}$ relationship is linear, it follows that the W_{LN} against $I_{s_{ur}}$ relationship is also linear, with these parameters inversely related by (see Appendix A also):

$$I_{s_{ur}} = 1 - W_{LN} \quad (9)$$

Hence s_{ur} values can be deduced for any water content w ($w_A < w < w_B$) from direct measurements of strength and corresponding water content performed on only two test-specimens, A and B , by

$$\log s_{ur} = (1 - W_{LN}) \left(\log \frac{s_{urA}}{s_{urB}} \right) + \log s_{urB} \quad (10)$$

Note that Eq. 8 may be expressed in terms of assigned strengths at the Atterberg limits (Eq. 11), and in this particular case, values of unity and zero for I_{sur} correspond to $s_{ur(PL)}$ and $s_{ur(LL)}$, respectively, and similarly for W_{LN} correspond to LL and PL respectively.

$$I_{sur} = \frac{\log(s_{ur}/s_{ur(LL)})}{\log R} \quad (11)$$

3. EXPERIMENTAL RESULTS AND DISCUSSION

Figures 4–8 present strengths for Casagrande LL and PL, along with values of R_s deduced from the database of 18 soils in Table 2.

Strength at LL

Considering first the data for mineral soils presented in Fig. 4a and excluding extreme values: deduced $s_{ur(LL)} = 0.64\text{--}2.10$ kPa (mean of 1.15 kPa, $n = 14$) for $LL = 30\text{--}402\%$. For the four organic sediments, deduced $s_{ur(LL)} = 0.86\text{--}0.98$ kPa (mean of 0.93 kPa) for $LL = 93\text{--}128\%$. These $s_{ur(LL)}$ values are within the reported range of 0.5–5.6 kPa for Casagrande LL (Wasti and Bezirci, 1985) and are consistent with the possible $s_{ur(LL)}$ range of 0.7–2.65 kPa suggested by Wroth and Wood (1978). Note that Youssef et al. (1965) and recent work by Haigh (2012a) suggest that the strength at LL may be a function of density and hence water content. However mean values of deduced $s_{ur(LL)}$ are considerably below $s_{ur(LL)} \approx 1.7$ kPa reported by Wroth and Wood (1978). Possible explanations may lie either in the residual operator sensitivity of the tests or in the choice of strength measurement/determination, with mobilized/deduced strengths in different apparatus dependent on a number of factors including shearing mode and rate, specimen confinement and boundary conditions (O’Kelly 2013b).

Also, although Sharma and Bora (2003) suggested a bi-linear $\log w$: $\log s_{ur}$ relationship occurs for montmorillonite soil at values of water content near LL, the $s_{ur(LL)}$ of 1.07 and 1.28 kPa deduced from regression analysis for the two bentonite soils considered in the present study were consistent with $s_{ur(LL)}$ values deduced for the 12 other mineral soils (Figs. 4a and 5).

Figure 4. Deduced strengths against liquid limit.

Figure 5. Deduced strengths at LL against water content.

The deduced $s_{ur(LL)}$ values are plotted against LL and plasticity index (IP) in Figs. 5 and 6a respectively. These data do not appear to show any strong relationship with either LL (plotted on a log scale after Youssef et al. (1965)) or IP , although no definitive conclusions can be drawn on account of the limited number of data points. In Fig. 7a, Casagrande LL values are compared against reported fall-cone LL values for the different soils. It appears that agreement between Casagrande and fall-cone results seems good for mineral soils having $LL < 140\%$. However, compared with fall-cone LL, Casagrande LL appears to define a higher value for LL in the case of the organic sediments and also mineral soils having $LL > 140\%$. Among the soils considered, the greatest deviation occurred for the two bentonite soils, B1 and B2 (Fig. 7a), with Casagrande LL of 334% and 402% respectively. Similar findings have been reported for mineral soils (Leroueil and Le Bihan, 1996, Farrell et al., 1997, Feng, 2001) and dredged organic sediments (Zentar et al., 2009). This agrees with recent work by Haigh (2012a) in that higher Casagrande LL would be predicted (versus fall-cone LL) for high LL soils as the density of high LL soils at LL will be low. The fall-cone is being used to determine a dynamic strength-related reference water content. The difference in LL values determined by the two approaches is also related to the behavior of clay under the different modes of deformation imposed by the different test apparatus, with soils having greater clay content (i.e., higher LL) likely to be greatly affected by viscoelastic effects (Budhu, 1985). Sridharan and Prakash (1999) reported that the mechanisms controlling undrained shear strength and liquid limit for kaolinitic soils is different from that of montmorillonitic soils and they concluded that the $s_{ur(LL)}$ value cannot be expected to be unique for all soils. However the link with mineralogy is still up for debate (Haigh, 2012b, Prakash, 2012).

Figure 6. Deduced strengths against plasticity index.

Figure 7. Comparison of Atterberg limits determined by Casagrande and fall-cone methods.

Strength at PL

Considering the data for mineral soils (Fig. 4b) and excluding extreme values, deduced $s_{ur(PL)}$ = 34–123 kPa (mean of 82 kPa, $n = 11$) for PL = 16–51%. Again, these values are within the reported ranges for $s_{ur(PL)}$ of 20–320 kPa (Kayabali and Tufenkci, 2010) and 17–530 kPa (Haigh et al. 2013). For the four organic sediments, deduced $s_{ur(PL)}$ = 9–26 kPa (mean of 17 kPa) for PL = 44–48%. The mean $s_{ur(PL)}$ values are considerably below $s_{ur(PL)} \approx 170$ kPa, considered by some researchers as typical of many soils. Again, a possible reason for this discrepancy may be due to the method of strength measurement/determination adopted by different researchers. The Casagrande thread-rolling method is also considered as a rather crude procedure (Belviso et al., 1985), often proving difficult to achieve reproducible PL values, unless the test is performed by an experienced person. Hence deduced values of $s_{ur(PL)}$ may naturally show a wide variation.

Furthermore, deduced $s_{ur(PL)}$ values do not appear to show any strong correlation with either LL or IP (Figs. 4b and 6b), although no definitive conclusions can be drawn on account of the limited number of data points. In Fig. 7b, the Casagrande PL values are compared against fall-cone PL_{100} values, which were also reported in the studies considered. From the available data, it would appear that the fall-cone approach predicts a lower value for PL and hence higher $s_{ur(PL)}$. This is particularly true for the four organic sediments, with the overestimation in $s_{ur(PL)}$ (compared to Casagrande PL) reaching about 20%. Similar findings have been reported by Feng (2000) and Kodikara et al. (2006). However a basic error is to assume that index properties obtained from the very different test procedures of the Casagrande and fall-cone methods should have more than a coincidental equivalence (Leroueil and Le Bihan, 1996). Casagrande PL is defined by the limit of workability of the soil thread during the rolling-out procedure and is therefore a reflection of both strength and deformability (i.e. soil toughness). Once one starts using cone tests for 'liquid limit' and 'plastic limit' then one has a completely different set of index properties, with the fall-cone PL_{100} defined in terms of a specific value of dynamic strength, which is 100 times the dynamic strength mobilized at the fall-cone LL.

Strength ratio R_s

Considering the data for mineral soils presented in Fig. 8, and excluding extreme values (i.e., taking upper end of R_s range as 170 (Sivakumar et al., 2009)), deduced $R_s = 43$ –128 with a mean of 75 for $n = 10$. This range is consistent with $R_s \approx 70$ reported for some Swedish clays by Whyte (1982). For the four organic sediments, deduced $R_s = 10$ –27 with a mean of 18. It has been observed that for amorphous organic residue material (OC = 57%) derived from the production of potable water from municipal works (O'Kelly, 2013a), the bulk material could still be remolded at water contents significantly below the measured Casagrande PL of 268%, at which a vane $s_{ur(PL)}$ of 30.5 kPa was mobilized. It is postulated that scale effects for the

thread-rolling method (Barnes and O’Kelly, 2011) may account for significantly lower Casagrande $s_{ur(PL)}$ and hence lower strength ratio R_s values for these organic sediments.

Figure 8. Strength ratio R_s against plasticity index.

Slight differences in measured values of water content from the actual LL and PL conditions can significantly affect the value of R_s , especially in the case of PL since shear strength increases exponentially with reducing water content (refer to Eq. 2). Numerous researchers (e.g. Belviso et al. (1985), Wood (1990) and Sivakumar et al. (2009)) have highlighted the relatively poor reproducibility of the thread-rolling method. This may account, to some degree, for deduced R_s values significantly greater or less than the value of 100 advocated by some researchers as typical for many soils. However, from Fig. 8, it would generally appear that actual values of R_s for different soils can be significantly greater or less than 100. Similar conclusions have been reported for dredged organic sediments by Zentar et al. (2009).

Hence the reliability of existing correlations, including Eqs. 1(a, g), 3 and 4, which assume a fixed value (usually 100) for the strength ratio is questionable, even in obtaining first approximation values. In contrast, the proposed water content–strength relationship given by Eq. 10 takes a different approach, with strength measurements performed on two plastic test-specimens instead of the Casagrande LL and PL tests. In other words, the proposed method does not rely on any tenuous link between the strengths mobilized at LL and PL, which for reasons described earlier in the paper, are in any case difficult to measure experimentally.

4. CONCLUSIONS

No unique value of undrained strength can be assigned either at the Casagrande LL or PL conditions for soils. Regression analysis of reported $\log w$: $\log s_{ur}$ correlations for different soils indicated that the strength ratio R_s can vary widely, with $R_s = 43$ –128 (mean of 75) for 14 mineral soils and $R_s = 10$ –27 (mean of 18) for four organic sediments considered. Deduced strengths at Casagrande LL and PL were within the range of values reported in literature, but the mean values were below $s_{ur(LL)} \approx 1.7$ kPa, $s_{ur(PL)} \approx 170$ kPa and $R_s \approx 100$ considered by some researchers as typical of many mineral fine-grained soils.

From the limited data, compared with fall-cone LL, Casagrande LL defined a higher value of water content for LL in the case of the organic sediments and also for mineral soils having $LL > 140\%$. This fits with recent work by Haigh (2012a), who suggested that the strength at LL may be a function of density and hence water content, which would predict higher Casagrande LL versus fall-cone LL since the density of high LL soils at LL will be low. . Furthermore, compared with fall-cone PL_{100} , Casagrande PL also defined a higher water content value for PL, and hence lower value of $s_{ur(PL)}$, especially for the organic sediments

where the underestimation compared to fall-cone PL_{100} reached about 20%. Deduced strengths at Casagrande LL and PL did not appear to show any strong relationship with either LL or IP .

Considering that the value of the strength ratio R_s is not the same for all soils, a new approach has been presented for predicting values of remolded undrained strength mobilized for different water contents within the plastic range. New parameters of relative water content W_{LN} and relative strength I_{sur} emerged from the well-established power law relationship between water content and strength. These indices are inversely related, and when introduced into the proposed relationship (Eq. 10), produce a reliable correlation for soils. In contrast, existing $\log w$: $\log s_{ur}$ correlations presuppose a value (usually 100) for the stress ratio, which as demonstrated in the present study and by other researchers, is generally not the case, especially for organic soils.

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Appendix A

Another justification of the proposed relationship given by Eq. 9 is presented here. From Eq. 4, one should work with ratios of water content rather than differences. Hence water content

ratio $R_w \left(= \frac{w_B}{w_A} \right)$ and strength ratio $R'_s \left(= \frac{s_{urA}}{s_{urB}} \right)$ are defined for test-specimens A and B ,

having respective strengths of s_{urA} and s_{urB} and water contents of w_A and w_B . Using Eqs. 7 and 8 for the newly defined parameters of relative water content W_{LN} and relative strength I_{sur} , and developing the relationship of $I_{sur} = 1 - W_{LN}$ gives:

$$\frac{\log(s_{ur}/s_{urB})}{\log(s_{urA}/s_{urB})} = 1 - \frac{\log w - \log w_A}{\log(w_B/w_A)}$$

$$\frac{\log(s_{ur}/s_{urB})}{\log R'_s} = 1 - \frac{\log w - \log w_A}{\log R_w}$$

$$\begin{aligned} \text{Hence, } \log w &= \log w_A + \log R_w + (\log s_{ur} - \log s_{urB}) \frac{\log R_w}{\log R'_s} \\ &= \log a - b \log s_{ur} \text{ (i.e., in the form of Eq. 2),} \end{aligned}$$

$$\begin{aligned} \text{with } \log a &= \log w_A + \log R_w - b \log s_{urB} = \log(w_B/s_{urB}^{-b}) \\ \text{and } b &= \log R_w / \log R'_s \text{ or } (w/w_A) = (s_{urA}/s_{ur})^b \end{aligned}$$

where a and b are soil-dependent coefficients, with a equal to the water content (in %) for $s_{ur} = 1$ kPa and b equal to the gradient of the $\log w$: $\log s_{ur}$ relationship (see Fig. 2). Hence the index property is given by $\log R_w / \log R'_s$; i.e. a comparison of corresponding water content and strength ratios.

TWO Tables:

Reference	Equation (s_{ur} , in kPa)	
Wroth and Wood (1978)	$s_{ur} = 170 \exp(-4.6 I_L)$	(1a)
Leroueil et al. (1983)	$s_{ur} = 1/(I_L - 0.21)^2$	(1b)
Locat and Demers (1988)	$s_{ur} = (19.8/I_L)^{2.44}; I_L > 1.0$	(1c)
Hirata et al. (1990)	$s_{ur} = \exp(-3.361 I_L + 0.376)$	(1d)
Terzaghi et al. (1996)	$s_{ur} = 2(I_L)^{-2.8}$	(1e)
Yilmaz (2000)	$s_{ur} = \exp(0.026 - 1.21 I_L)$	(1f)
Koumoto and Houlsby (2001)	$s_{ur_{FC}} = \exp[(1.070 - I_{LN})/0.217]$	(1g)
NGI (2002)	$s_{ur} = 4.2(I_L)^{-1.6}$	(1h)
NGI (2002)	$s_{ur} = 3.9(I_L)^{-2.0}$	(1i)
Yang et al. (2006)	$s_{ur} = 159.6 \exp(-3.97 I_L)$	(1j)

Table 1. Selection of published strength–water content correlations for remoulded soils. Note: Eq. 1a is based on $s_{ur(LL)} = 1.7$ kPa and $R_s = 100$; Eq. 1g is based on a dynamic $s_{ur(LL)} = 1.38$ kPa mobilised for the 60g–60° fall cone, with LL defined by a cone penetration depth of 11.5 mm and assuming $R_s = 100$.

Soil name	LL (%)	PL (%)	Organic content (%)	Coefficient <i>a</i>	Coefficient <i>b</i>	Regression coefficient	Number of data points	Strength test	Reference
Horton	30	16	0	–	–	–	nr	nr	Skempton & Northey (1953)
London	73	25	0	–	–	–	nr	nr	
Shellhaven	97	32	0	–	–	–	nr	nr	
Yagusa clay, Y	66.7	28.6	0	27.26	0.200	0.997	10		Koumoto & Houlby (2001)
Ariake clay, A1	120.0	51.2	0	60.51	0.156	0.994	5		
Ariake clay, A2	130.9	51.0	0	39.24	0.242	0.990	7		
Ariake clay, A3	140.0	54.0	0	29.54	0.326	0.980	nr		
Clay mixture, M1	74.0	36.0	0	44.34	0.172	1.000	nr	Lab	
Clay mixture, M2	178.0	40.6	0	39.97	0.341	0.999	6	vane	
Clay mixture, M3	180.0	51.0	0	38.14	0.336	0.990	nr		
Clay mixture, M4	190.5	35.6	0	38.82	0.412	0.998	9		
Bentonite, B1	334.2	43.3	0	32.45	0.535	0.970	nr		
Bentonite, B2	402.0	35.6	0	31.72	0.560	0.995	11		
Kaolin	37.5	23.7	0	42.56	0.201	0.998	6		Zentar et al. (2009)
F5	128.3	48.4	9.7	127.54	0.296	0.994	6	Lab	
F6	113.9	47.2	9.3	112.74	0.292	0.993	6	vane	
F12	101.9	46.4	7.0	97.68	0.289	1.000	5		
F13	93.0	43.6	6.7	90.27	0.330	0.997	5		

Table 2. Database of Atterberg limits and strength coefficients *a* and *b*. Note: nr, not reported.

EIGHT Figures:

Figure 1. Liquidity index against remolded undrained strength (adapted from Koumoto and Houslyb (2001), Géotechnique Vol. 51, No. 8, pp. 701–712).

Figure 2. Water content (in %) against remolded undrained strength (in kPa) for saturated soil. Note: CSL , projection of the critical-state line.

Figure 3. Modified liquidity index against remolded undrained strength (adapted from Koumoto and Houlsby (2001), *Géotechnique* Vol. 51, No. 8, pp. 701–712).

(a) Strength at *LL*.

(b) Strength at *PL*.

Figure 4. Deduced strengths against liquid limit.

Figure 5. Deduced strengths at LL against water content.

(a) Strength at LL .

(b) Strength at PL .

Figure 6. Deduced strengths against plasticity index.

(a) At *LL*.

(b) At *PL*.

Figure 7. Comparison of Atterberg limits determined by Casagrande and fall-cone methods.

Figure 8. Strength ratio R_s against plasticity index.