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Notch and Mean Stress Effect in Fatigue as Phenomena of Elasto-Plastic Inherent Multiaxiality

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ABSTRACT

The present paper summarises an attempt of estimating fatigue lifetime of notched metallic materials by directly accounting for the degree of multiaxiality of the local elasto-plastic stress/strain fields acting on the fatigue process zone. In more detail, the proposed approach takes as its starting point the assumption that Stage I is the most important stage to be modelled to accurately estimate fatigue damage, and this holds true independently of the sharpness of the assessed geometrical feature. According to this initial idea, and by taking full advantage of the so-called Modified Manson-Coffin Curve Method (MMCCM), the hypothesis is then formed that the crack initiation plane is always coincident with that material plane experiencing the maximum shear strain amplitude. Subsequently, to devise an efficient design method capable of taking into account the detrimental effect of stress/strain gradients arising also from severe stress/strain concentration phenomena, the MMCCM is suggested here as being applied in terms of the Theory of Critical Distances (TCD), the latter being used in the form of the Point Method (PM). Further, in light of the well-known fact that the value of the mean stress/strain components in the vicinity of the stress/strain raisers' apices can be different from the corresponding nominal values due to the actual elasto-plastic behaviour of the material being assessed, it is shown, through the MMCCM itself, that also the mean stress effect can directly and accurately be treated as a problem of inherent multiaxiality. Finally, as a preliminary validation, the accuracy and reliability of the proposed approach is checked through several experimental results taken from the literature and generated by testing, under uniaxial fatigue loading, samples containing a variety of geometrical features, the effect of different nominal load ratios being investigated as well.

Keywords: Critical plane approach, Theory of Critical Distances, cyclic plasticity, notch, multiaxial stress/strain fields

NOMENCLATURE

b	Fatigue Strength Exponent
b_0	Shear Fatigue Strength Exponent
c	Fatigue Ductility Exponent
c_0	Shear Fatigue Ductility Exponent
m	Mean stress sensitivity index
n'	Cyclic Strain Hardening Exponent
r	Notch root radius
K'	Cyclic Strength Coefficient
K_t	Net stress concentration factor
K_{tg}	Gross stress concentration factor
L_γ	Multiaxial critical distance value
N_f	Number of cycles to failure
$N_{f,e}$	Estimated number of cycles to failure
O_{xyz}	Frame of reference
R	Load ratio ($R=\sigma_{min}/\sigma_{max}$)
R_ϵ	Strain ratio ($R_\epsilon=\epsilon_{min}/\epsilon_{max}$)
ϵ'_f	Fatigue Ductility Coefficient
$\epsilon_x, \epsilon_y, \epsilon_z$	Normal strains
δ	Out-of-phase angle
γ_f	Shear Fatigue Ductility Coefficient
$\gamma_f(\rho)$	Multiaxial Fatigue Ductility Coefficient depending on ρ
γ_a	Shear strain amplitude relative to the critical plane
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	shear strains
ν_e	Poisson's ratio for elastic strain
ν_{eff}	Effective value of Poisson's ratio
ν_p	Poisson's ratio for plastic strain
σ'_f	Fatigue Strength Coefficient
σ_n	Stabilised stress perpendicular to the critical plane
$\sigma_{n,max}$	Stabilised Maximum stress perpendicular to the critical plane
σ_{nom}	Nominal stress
$\sigma_x, \sigma_y, \sigma_z$	Normal stresses
σ_y	Yield stress
σ_{UTS}	Ultimate tensile strength
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	Shear stresses
ρ	Stress ratio relative to the critical plane
ρ_{eff}	Effective value of the stress ratio relative to the critical plane
τ_a	Stabilised shear stress amplitude relative to the critical plane
τ'_f	Shear Fatigue Strength Coefficient
$\tau'_f(\rho)$	Multiaxial Fatigue Strength Coefficient depending on ratio ρ
Δ	Range

Subscripts

a	amplitude
m	mean value
max	max value
min	min value

1. INTRODUCTION

Multiaxial fatigue and fracture is usually treated as a problem involving external systems of complex forces and moments resulting in multiaxial stress/strain states that damage engineering materials' critical locations: a classical example is a shaft subjected to combined bending and torsion. It is the writers' opinion that the above situations are instead just sub-cases of the more complex multiaxial fatigue issue: in fact, multiaxial fatigue involves not only external but also inherent, i.e. internal, multiaxiality [1]. This firm belief is supported by the well-known fact that also in a sample subjected to uniaxial cyclic loading the presence of a notch results, in the vicinity of the stress raiser's apex, in local cyclic stress/strain fields which are always, at least, biaxial [1-4]. The relevant peculiarity which makes such a problem somehow easier to be addressed in situations of practical interest is not the fact that the applied forces are uniaxial (i.e., either axial or bending loading), but the fact that the local stress/strain fields always vary proportionally (i.e., the resulting stress/strain components are in-phase). This should explain the reason why, as long as notched materials are subjected to external uniaxial time-dependent loading, fatigue lifetime can always be estimated accurately by considering only the maximum principal stress or strain components: since the relevant stresses and strains in the fatigue process zone are in-phase, either σ_1 or ϵ_1 are in any case representative of the entire stress/strain field distribution.

Owing to the scenario described above, it is logical to presume then that, given both the material and the degree of multiaxiality of the local stress/strain fields, the resulting fatigue damage has to be the same independently of the source the multiaxiality itself arises from [1]. Accordingly, in what follows an attempt is made to address the problem of estimating fatigue damage resulting from inherent multiaxiality by employing an elasto-plastic critical plane approach, i.e., the MMCCM, which was originally devised and validated by considering situations involving solely external multiaxiality [5, 6].

Since, as said above, the present paper aims to investigate the problem of estimating fatigue lifetime of notched components by explicitly modelling the cyclic elasto-plastic behaviour of the material

being assessed, another important aspect which definitely deserves to be recalled here briefly is that, according to the classical strain based approach, it should always be possible to evaluate fatigue damage in notched components by directly using the elasto-plastic root stresses and strains [7]. In spite of the practical difficulties always encountered when calculating the above quantities (and this holds true not only when the problem is addressed using numerical approaches, but also when it is addressed using classical analytical solutions, like, for instance, the well-known rule devised by Neuber [8, 9]), nowadays the conventional strain-life method is daily used by engineers engaged in designing real components against fatigue. From a reliability and safety point of view, the usage of the elasto-plastic root stresses and strains to perform the fatigue assessment of notched metallic materials is seen to result in estimates which are, in general, characterised by a certain degree of conservatism, where such a degree of conservatism increases as the sharpness of the notch itself increases [10, 11].

Examination of the state of the art shows that an efficient way to improve the strain based approach's accuracy in estimating fatigue damage in those components weakened by relatively sharp notches may be adopting appropriate elasto-plastic fatigue strength reduction factors [10]. In other words, the detrimental effect of severe stress/strain gradients can directly be accounted for by following a strategy similar to the one proposed by Neuber [12] and Peterson [13] to estimate notch fatigue strength in the high-cycle fatigue regime. In this scenario, it is worth mentioning here that Susmel and Taylor [14] have recently proven that Peterson's Point Method as well as Neuber's Line Method can successfully be extended back to the low/medium cycle fatigue regime by simply modelling metallic materials' cyclic elasto-plastic behaviour explicitly. The main advantage of such a *modus operandi* is that the critical distance value is no longer dependent on the number of loading cycles to failure as it happens instead when the Theory of Critical Distances (TCD) is applied by post-processing linear-elastic stress fields [15].

The last aspect of the problem which has to be considered in great detail here is the effect of superimposed static stresses and strains on the overall fatigue strength of engineering materials. In

more detail, it was seen from the experiments that, as far as plain materials are involved, in general the presence of non-zero mean strain can be disregarded due to the mean stress relaxation phenomenon [7]: only when the mean stress is not fully recovered, mean strains have a detrimental effect on the overall fatigue strength of the material being assessed. Further, the mean stress relaxation phenomenon is seen to be more pronounced in the low-cycle rather than in the high-cycle fatigue regime, so that, the importance of the role played by the presence of superimposed static stresses is seen to increase with increasing of the number of loading cycles to failure. In the presence of stress/strain concentration phenomena, the problem of correctly modelling the mean stress effect is further complicated by the fact that, in the vicinity of the notch apex, the local value of the strain ratio, calculated in terms of elasto-plastic deformations, depends not only on the load ratio characterising the applied nominal loading but also on the elasto-plastic behaviour of the considered material as well as on the features of the assessed stress/strain raiser. Further, given both the geometrical feature and the material properties, such a local strain ratio can vary as the magnitude of the applied nominal loading changes. The most evident implication of such a complex mutual interaction among different variables is that the local value of the strain ratio can be larger than -1 also when the component being assessed is subjected to fully-reversed uniaxial nominal fatigue loading.

To conclude, it can be pointed out that, in this complex scenario, the aim of the theoretical work summarised in the present paper is to formalise an alternative lifetime estimation technique based on the assumption that both the notch and mean stress effect in fatigue can efficiently be treated as phenomena of elasto-plastic inherent multiaxiality.

2. FATIGUE DAMAGE MODEL

In order to formalise an efficient design method capable of correctly taking into account the degree of multiaxiality of the elasto-plastic stress/strain fields damaging the assumed critical locations, the first logical step is, of course, the adoption of a consistent fatigue damage model.

In order to fully address such a crucial aspect, initially it is worth recalling here that, as far as plain materials subjected to uniaxial fatigue loading are concerned, the micro/meso-crack propagation phenomenon is usually subdivided into two different phases [16]. In more detail, Stage I cracks propagate on those crystallographic planes of maximum shear and their growth is Mode II dominated. On the contrary, Stage II cracks, which take over from the initial Stage I propagation, tend to orient themselves in order to experience the maximum Mode I loading, i.e., the maximum opening cyclic stress [16]. According to the above schematisation, Stage I is assumed to be controlled by the microscopic shear acting on to those easy glide planes experiencing the maximum shear. Further, the length of Stage I cracks is seen to depend not only on the local microstructural features of the material, but also on the magnitude of the applied cyclic loading: generally speaking, the maximum length of Stage I micro/meso-cracks is of the order of a few grains and, given the material, the Stage I process seems to be more predominant in the low- rather than in the high-cycle fatigue regime [17].

As mentioned above, the Stage I propagation is followed by a Stage II growth that occurs due to *“plastic de-cohesion on the planes of maximum shear strain gradient at the crack tip”* [18], and, according to Tomkins, *“the same mechanism is operative also in Stage I growth, but de-cohesion occurs on only one of the available shear planes”* [18].

The same considerations apply also to those situations involving external multiaxial fatigue loading. In particular, initially it is worth noticing here that, when plain metallic materials are subjected to multiaxial cyclic loading, micro/meso-cracks can propagate either on the component surface or inwards, the latter situation being the most damaging one [19, 20]. As to the Stage I propagation under multiaxial fatigue loading, in Ref. [21] Kanazawa, Miller and Brown affirmed: *“Stage I cracks form on crystallographic planes, being slip planes within individual grains of metal. These are not necessarily the planes of maximum shear in the macroscopic sense, but rather the slip system most closely aligned to these planes. Clearly, the slip systems which experience the greatest amount of deformation are those which align precisely with the maximum shear direction, and*

therefore most fatigue cracks initiate in these grains. But slip systems with lesser degrees of shear also initiate cracks at a slower rate”.

According to the above experimental outcomes, the hypothesis can be formed then that, given the assumed crack initiation location (point O in Figure 1a), fatigue damage reaches its maximum value on that material plane experiencing the maximum shear strain amplitude, γ_a [1, 5, 6] (see Figure 1b), and it holds true independently of the complexity of the loading path damaging the material being assessed. Further, it is hypothesised that, in order to correctly take into account the mean stress effect, according to Socie [22], not only the amplitude, $\sigma_{n,a}$, but also the mean value, $\sigma_{n,m}$, of the stress normal to the critical plane has to be incorporated into the fatigue damage model (Fig. 1b). In order to define the effective value of the stress ratio relative to the critical plane, the combined effect of $\sigma_{n,a}$ and $\sigma_{n,m}$ can efficiently be taken into account by weighing them through the stabilised shear stress, τ_a , relative to the critical plane itself, that is [1, 5, 23]:

$$\rho_{\text{eff}} = \frac{m \cdot \sigma_{n,m} + \sigma_{n,a}}{\tau_a} \quad (1)$$

where m is the so-called *mean stress sensitivity index*, that is, a material constant varying in the range 0 to 1. In more detail, when $m=0$ the material is assumed to be fully insensitive to the presence of superimposed static stresses, whereas when $m=1$ the assessed material is fully sensitive to the mean stress perpendicular to the critical plane. The most important feature of ρ_{eff} is that the value of such a stress ratio is seen to depend not only on the magnitude of the superimposed static stresses, but also on the degree of multiaxiality and non-proportionality of the stress/strain state damaging the assumed critical point [1, 5]. Another interesting attempt to take into account the combined effect of $\sigma_{n,a}$ and $\sigma_{n,m}$ is discussed in great detail in Ref. [24].

As to the effect of the static stresses perpendicular to the critical plane, it is worth noticing here that Kaufman and Topper [25] have proven that, as soon as the mean normal stress relative to the critical

plane becomes larger than a certain material threshold value, a further increase of such a stress component does not result in any further increase of fatigue damage. They have explained the above experimental evidence by observing that, as long as $\sigma_{n,m}$ is lower than the aforementioned material threshold value, the effect of the shearing forces pushing the tips of the micro/meso crack is reduced because of the existing interactions amongst the morphological asperities characterising the two faces of the crack itself: this should make it clear that this additional friction inevitably results in a decrease of the crack growth rate. On the contrary, as long as micro/meso cracks are fully open, the shearing forces are transmitted to the crack tips in full, resulting in a Mode II propagation which is affected by no additional frictional phenomena. This implies that, under these circumstances, given the magnitude of the shear forces, a further increase of the normal mean stress relative to the critical plane does not result in any further acceleration of the crack propagation phenomenon.

From the above considerations, it can be realised that the fatigue damage model adopted in the present investigation is based on the following three fundamental hypotheses (Fig. 1b): Stage I cracks are assumed to form on that material plane (i.e., the so-called critical plane) experiencing the maximum shear strain amplitude, γ_a ; the amplitude of the stabilised stress perpendicular to the critical plane, $\sigma_{n,a}$, favours the propagation phenomenon by cyclically opening and closing micro/meso cracks; finally, the portion of the stabilised mean stress normal to the critical plane that effectively contributes to the initiation and propagation phenomenon is equal to $m \cdot \sigma_{n,m}$, where m is a material constant to be determined experimentally.

To conclude, it is worth observing that, as far as conventional engineering materials are concerned, m can be taken equal to unity [5, 6]. In particular, for $m=1$ Eq. (1) simplifies into:

$$\rho = \frac{\sigma_{n,m} + \sigma_{n,a}}{\tau_a} = \frac{\sigma_{n,max}}{\tau_a} \quad (2)$$

In other words, according to Eq. (2), the mean stress effect in fatigue is taken into account as suggested by Socie in Ref. [22]: it is evident that this assumption results in a great simplification of the design problem, so that, in what follows, fatigue damage is simply estimated by taking full advantage of Eq. (2). At the same time, in the presence of certain particular engineering materials, the systematic usage of a mean stress sensitivity index equal to unity may result in estimates which are characterised by an excessive degree of conservatism [1]: in these circumstances, the accuracy of the predictions can be increased by simply running appropriate experiments to determine m .

3. FUNDAMENTALS OF THE MODIFIED MANSON-COFFIN CURVE METHOD

After discussing the fatigue damage model on which the present paper is based, the subsequent step is selecting an appropriate criterion capable of estimating fatigue lifetime under external multiaxial loading paths. As said above, thanks to its particular features, the fatigue lifetime estimation technique formalised in what follows takes full advantage of the so-called Modified Manson-Coffin Curve Method (MMCCM) [1, 5, 6]. Before reviewing the main features of the MMCCM, it is worth observing here that such a criterion represents nothing but an elasto-plastic reformulation of the so-called Modified Wöhler Curve Method, that is, a non-conventional bi-parametrical linear-elastic critical plane approach we have devised and validated in recent years [1, 26, 27]. As to the way the above two methods work, it is important to observe here that, even though their formalisation was based on a similar reasoning, the fact that the MMCCM is designed to explicitly take into account the elasto-plastic behaviour of engineering materials results in governing equations which lead to slightly different estimates when the two criteria are used to estimate fatigue damage in the elastic regime [1, 5].

The MMCCM postulates that fatigue lifetime can accurately be evaluated by simply using non-conventional bi-parametrical Manson-Coffin curves [5]. In more detail, such an approach takes as its starting point the idea that fatigue damage can be summarised in log-log diagrams plotting the

shear strain amplitude, γ_a , relative to that plane experiencing the maximum shear strain amplitude (i.e., the so-called critical plane) against the number of reversals to failure, $2N_f$ (Fig. 2a). As to the orientation of the critical plane, it is worth observing here that, in principle, any of the available definitions (see Ref. [1] and references reported therein) can efficiently be used to locate that material plane experiencing the maximum shear strain amplitude. However, it is the writers' opinion that, amongst those methods which have specifically been devised to address such a tricky problem, the Longest Chord Method [28] is not only very simple to use but also very effective when applied in conjunction with the critical plane concept.

Turning back to the formalisation of the MMCCM, by using a large number of experimental results generated under proportional and non-proportional multiaxial loading paths [5], it was shown that modified Manson-Coffin curves shift downwards in the corresponding γ_a vs. $2N_f$ diagram as the value of ratio ρ increases (Fig. 2a). In other words, the MMCCM postulates that, given the shear strain amplitude relative to the critical plane, fatigue damage increases with increasing of the $\sigma_{n,max}$ to τ_a ratio. According to the schematic chart reported in Figure 2a, the equation describing any Modified Manson-Coffin curve can directly be expressed as follows:

$$\gamma_a = \frac{\tau'_f(\rho)}{G} (2N_f)^{b(\rho)} + \gamma'_f(\rho) \cdot (2N_f)^{c(\rho)} \quad (3)$$

where $\tau'_f(\rho)$, $\gamma'_f(\rho)$, $b(\rho)$ and $c(\rho)$ take on the following form [5]:

$$\frac{\tau'_f(\rho)}{G} = \rho \cdot (1 + \nu_e) \frac{\sigma'_f}{E} + (1 - \rho) \frac{\tau'_f}{G}; \quad b(\rho) = \frac{b \cdot b_0}{(b_0 - b)\rho + b} \quad (4)$$

$$\gamma'_f(\rho) = \rho \cdot (1 + \nu_p) \epsilon'_f + (1 - \rho) \gamma'_f; \quad c(\rho) = \frac{c \cdot c_0}{(c_0 - c)\rho + c} \quad (5)$$

By following a fairly articulated reasoning, such calibration functions were derived by directly using the conventional fully-reversed uniaxial and torsional Manson-Coffin fatigue curves rewritten in terms of Tresca's equivalent stress, that is [1, 5]:

$$\gamma_a = (1 + \nu_e) \frac{\sigma'_f}{E} (2N_f)^b + (1 + \nu_p) \epsilon'_f (2N_f)^c \quad (\text{Uniaxial case, } \rho=1) \quad (6)$$

$$\gamma_a = \frac{\tau'_f}{G} (2N_f)^{b_0} + \gamma'_f (2N_f)^{c_0} \quad (\text{Torsional case, } \rho=0) \quad (7)$$

where ν_e and ν_p are Poisson's ratio for elastic and plastic strain, respectively.

To conclude, it is worth observing here that, as proven in Ref. [5], the systematic use of the MMCCM to estimate a large number of experimental results, taken from the literature and generated by testing, under strain control, a variety of plain metallic materials under proportional and non-proportional multiaxial loading paths, was seen to result in highly accurate predictions, that is, in estimates falling within an error factor of about 3 (Fig. 2b), the error factor being defined as the ratio between experimental and estimated fatigue lifetime. Local stresses and strains were evaluated by means of FE analyses by assuming Von Mises plasticity rule and isotropic hardening behaviour of the material. Lastly, as shown in the experimental, N_f , vs. estimated, $N_{f,e}$, fatigue lifetime diagram of Figure 2b, the MMCCM proved to be capable of correctly taking into account also the presence of superimposed static stresses.

4. MEAN STRESS EFFECT VS. INHERENT MULTIAXIALITY

As said at the end of the previous Section, the MMCCM is seen to be highly accurate also in estimating the detrimental effect of non-zero mean stresses and strains. Such a remarkable accuracy may be explained by forming the hypothesis that the mean stress effect in fatigue is nothing but a problem of inherent multiaxiality, so that, thanks to its nature, the MMCCM is fully sensitive to the presence of superimposed static stresses and strains.

In more detail, consider the sample sketched in Figure 1a and assume that it is subjected to a cyclic axial strain characterised by a strain ratio, $R_\epsilon = \epsilon_{x,\min}/\epsilon_{x,\max}$, larger than -1, such a situation being described in terms of Mohr's circles in Figure 3a. According to the specific stabilised elasto-plastic behaviour of the investigated material, Mohr's circles representing the corresponding stress state are those sketched in Figure 3b.

As to the situation depicted in Figure 3, it is important to remember that, in general, R_ϵ is seen to be different from the corresponding load ratio, $R = \sigma_{x,\min}/\sigma_{x,\max}$, due to well-known phenomena like stress/strain hardening and softening, cyclic creep and, above all, mean stress relaxation. In particular, it is evident that such a schematisation applies solely to those situations in which the mean stress due to the presence of a non-zero axial mean strain is not fully recovered in the stabilised regime. Another important fact is that, given the material, even though the strain ratio, R_ϵ , is kept constant, the corresponding load ratio R is seen to vary as the amplitude of the induced deformation increases. In a similar way, under a constant value of R , the resulting ratio R_ϵ changes as the amplitude of the applied stress increases. As to the latter scenario, it is straightforward to see that, under force controlled uniaxial loading, ratio ρ directly depends on the applied load ratio, R [1]. In more detail, in the presence of a superimposed axial static stress, it is trivial to obtain the following identities allowing the relevant stress components relative to the critical plane to be calculated directly, i.e. [1]:

$$\begin{aligned}\tau_a &= \frac{\sigma_{x,\max}}{4}(1-R) = \frac{\sigma_{x,a}}{2} \\ \sigma_{n,\max} &= \frac{\sigma_{x,\max}}{2} = \frac{\sigma_{x,m} + \sigma_{x,a}}{2} \Rightarrow \sigma_{x,m} = 2(\sigma_{n,\max} - \tau_a) \\ \sigma_{n,m} &= \frac{\sigma_{x,m}}{2}; \sigma_{n,a} = \frac{\sigma_{x,a}}{2}\end{aligned}$$

which yield to the following closed-form expression for ρ [1]:

$$\rho = \frac{\sigma_{n,\max}}{\tau_a} = \frac{2}{(1-R)} \quad (8)$$

The above identity, together with Figure 4a, should make it evident that, in stress controlled uniaxial situations, ratio ρ increases as the load ratio, R , of the applied cyclic loading increases (case $\tau_{xy,a}/\sigma_{x,a}=0$ in Figure 4a). In a similar way, ratio ρ is seen to vary as R changes also in the presence of multiaxial fatigue loading. For instance, as shown in Figure 4a, under in-phase axial (or bending) and torsional loading (i.e., by taking the out-of-phase angle equal to zero), given the $\tau_{xy,a}$ to $\sigma_{x,a}$ ratio, stress index ρ increases with increasing of $R=\sigma_{x,\min}/\sigma_{x,\max}=\tau_{xy,\min}/\tau_{xy,\max}$. Further, the $\sigma_{n,\max}$ to τ_a ratio is also highly sensitive to the degree of non-proportionality of the applied loading path: for instance, as shown in Figure 4b, under fully-reversed out-of-phase axial and torsional loading, ρ is always maximised for a value of the shift angle, δ , equal to 90° , and this holds true independently of the ratio between the amplitudes of the two nominal stress components.

Even though all the considerations reported in the previous paragraphs refer to situations where the assessed material exhibits a purely elastic behaviour, similar conclusions can be drawn also when cyclic plasticity is taken into account: the only difference is that the existing relationships amongst ρ , R and R_ε fully depend on the actual elasto-plastic behaviour of the investigated material, where, in the presence of out-phase loadings, also the additional non-proportional hardening plays a fundamental role [29-32].

In brief, according to the reasoning summarised in the present Section, it is possible to argue that, since ρ is a stress index capable of measuring, in an engineering way, the degree of multiaxiality of the stress/strain state damaging the assumed critical points [1, 5], the mean stress effect in fatigue

can efficiently be treated as a problem of inherent multiaxiality simply because an increase of either R or R_e results in an increase of the ρ ratio itself.

To further support the validity of the above assumption, the experimental, N_f , vs. estimated, $N_{f,e}$, fatigue lifetime diagram reported in Figure 5 shows the accuracy of the MMCCM in taking into account the mean stress effect in plain samples subjected to strain controlled axial loading, the static and fatigue properties of the considered materials being listed in Table 1. As to the data summarised in the above chart, it is worth observing that, for any experimental point, the amplitude and the mean value of the stabilised stress were always gathered during testing. Further, the fully-reversed torsional Manson-Coffin curve was available both for carbon steel SAE 1045 [33] and for nickel-chromium alloy Inconel 718 [34], whereas such a curve was estimated for low-carbon steel En3B according to von Mises [29]. The diagram in Figure 5 fully confirms that the MMCCM is highly accurate in estimating fatigue damage in the presence of superimposed static stresses by simply treating the mean stress effect as a problem of elasto-plastic inherent multiaxiality.

At this point, it is interesting to compare the accuracy of the MMCCM in taking into account the presence of superimposed axial stresses to the one of those classical criteria commonly used in situations of practical interest to design real components against fatigue. In particular, the following three well-known strain based relationships are considered:

$$\varepsilon_{x,a} = \frac{\sigma'_f - \sigma_{x,m}}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (\text{the Morrow formula [35]}) \quad (9)$$

$$\varepsilon_{x,a} = \frac{\sigma'_f - \sigma_{x,m}}{E} (2N_f)^b + \varepsilon'_f \left(\frac{\sigma'_f - \sigma_{x,m}}{\sigma'_f} \right)^{\frac{c}{b}} (2N_f)^c \quad (\text{the modified Morrow formula [36]}) \quad (10)$$

$$\varepsilon_{x,a} \sigma_{x,\max} = \frac{\sigma'^2_f}{E} (2N_f)^{2b} + \sigma'_f \varepsilon'_f (2N_f)^{b+c} \quad (\text{the SWT parameter [37]}) \quad (11)$$

The N_f vs. $N_{f,e}$ diagram in Figure 6a makes it evident that, when applied to the same data as those summarised in Figure 5 as well as in Table 1, the use of the MMCCM results in the same level of accuracy as the one obtained by applying the classical methods, i.e., Eqs (9) to (11): this outcome is definitively remarkable, since the formalisation of the MMCCM is based on assumptions different from those adopted by Morrow as well as by Smith, Watson and Topper to devise their well-known methods.

In order to further deepen the investigation on the MMCCM's accuracy in modelling the mean stress effect in fatigue, for the sake of completeness, it is interesting also to compare our method's estimates to the ones obtained from those stress-based parameters specifically devised to estimate the detrimental effect of superimposed static stresses. In particular, as reported by Dowling [38], to estimate finite lifetime in the presence of non-zero mean stresses, in the classical criteria the endurance limit can directly be replaced with the right-hand side of Basquin's relationship [39], obtaining [38, 40, 41]:

$$\frac{\sigma_{x,a}}{\sigma'_f (2N_f)^b} + \frac{\sigma_{x,m}}{\sigma_y} = 1 \Rightarrow \sigma_{x,a} = \sigma'_f (2N_f)^b \left(1 - \frac{\sigma_{x,m}}{\sigma_y} \right) \quad (\text{Soderberg's formula}) \quad (12)$$

$$\frac{\sigma_{x,a}}{\sigma'_f (2N_f)^b} + \frac{\sigma_{x,m}}{\sigma_{UTS}} = 1 \Rightarrow \sigma_{x,a} = \sigma'_f (2N_f)^b \left(1 - \frac{\sigma_{x,m}}{\sigma_{UTS}} \right) \quad (\text{Goodman's formula}) \quad (13)$$

$$\frac{\sigma_{x,a}}{\sigma'_f (2N_f)^b} + \left(\frac{\sigma_{x,m}}{\sigma_{UTS}} \right)^2 = 1 \Rightarrow \sigma_{x,a} = \sigma'_f (2N_f)^b \left[1 - \left(\frac{\sigma_{x,m}}{\sigma_{UTS}} \right)^2 \right] \quad (\text{Gerber's parabola}) \quad (14)$$

$$\left(\frac{\sigma_{x,a}}{\sigma'_f (2N_f)^b} \right)^2 + \frac{\sigma_{x,m}}{\sigma_{UTS}} = 1 \Rightarrow \sigma_{x,a} = \sigma'_f (2N_f)^b \sqrt{1 - \frac{\sigma_{x,m}}{\sigma_{UTS}}} \quad (\text{Dietman's parabola}) \quad (15)$$

$$\left(\frac{\sigma_{x,a}}{\sigma'_f (2N_f)^b}\right)^2 + \left(\frac{\sigma_{x,m}}{\sigma_{UTS}}\right)^2 = 1 \Rightarrow \sigma_{x,a} = \sigma'_f (2N_f)^b \sqrt{1 - \left(\frac{\sigma_{x,m}}{\sigma_{UTS}}\right)^2} \quad (\text{Elliptical relationship}) \quad (16)$$

To conclude, the N_f vs. $N_{f,e}$ diagram reported in Figure 6b shows that, again for the data summarised in Figure 5 and Table 1, compared to the MMCCM, the systematic usage of the above classical stress-based formula resulted in estimates characterised by an evident degree of conservatism.

5. POINT METHOD AND STRUCTURAL VOLUME

As briefly recalled above, the classical strain-based approach postulates that fatigue damage in components containing geometrical features can directly be estimated through notch-root stresses and strains [7]. Unfortunately, such a *modus operandi*, even if very appealing from an engineering point of view, results in estimates whose degree of conservatism is seen to increase as the sharpness of the assessed notch increases [11]. In order to overcome the above problem, Susmel and Taylor [14] have recently proven that fatigue lifetime of notched components can efficiently be estimated by addressing the problem in terms of the Theory of Critical Distances (TCD) applied in the form of both the Point (PM) and Line Method (LM). In more detail, in the above preliminary investigation fatigue damage was estimated by post-processing the stabilised elasto-plastic stress/strain fields damaging the material in the vicinity of stress/strain raisers' apices according to the maximum principal stress/strain criterion, i.e. by neglecting the actual degree of multiaxiality of the stress/strain state at the assumed critical locations.

In order to reformulate the above idea according to the fatigue damage model adopted in the present study, consider then a notched component subjected to an external system of cyclic forces resulting in a multiaxial stress/strain field acting on the material portion close to the notch tip (Fig. 7). It is

worth recalling here that, according to Kanazawa, Miller and Brown [21], the formation of Stage I cracks depends on the micro-stress/strain components relative to those slip planes most closely aligned to the macroscopic material planes experiencing the maximum shear strain amplitude. Since, in situations of practical interest, the actual orientation of such crystallographic planes is never known, by using the Volume Method argument [2, 8, 42, 43] the hypothesis can be formed that a macroscopic stress/strain state representative of the fatigue damage extent in those grains situated in the vicinity of the notch apex can be estimated by simply averaging the elasto-plastic stress over the fatigue process zone (i.e., over the so-called structural volume). Further, according to the TCD (see Ref. [2] and references reported therein), the reference stress/strain state determined in terms of the Volume Method is the same as that determined at a given distance from the stress concentrator apex (Fig. 7), i.e., calculated according to the PM.

If the stress/strain state determined at the centre of the structural volume is assumed then to give an engineering information representative of the average microscopic stress/strain states damaging those grains located in the vicinity of the crack initiation site, it can then be hypothesised that the shear and normal macroscopic stresses and strains relative to the critical plane are somehow related to the corresponding microscopic quantities acting on the most damaged glide planes (Fig. 7). According to the above idea, fatigue strength not only in notched, but also in plain engineering materials, may then be estimated by considering the stabilised stress/strain state at a distance from the assumed crack initiation site equal to $L_\gamma/2$, L_γ being treated as a material constant. The in-field procedure suitable for determining critical distance L_γ as well as the validity of this hypothesis will be discussed below in great detail.

In order to somehow investigate the existing link between the idea sketched in Figure 7 and experimental reality, Figure 8 shows two micrographs of the cracked material in the vicinity of the notch tips of two samples made of low-carbon steel C10 and tested under zero-tension uniaxial fatigue loading [44]. The above pictures show that, according to Tomkins [18] and Miller [45, 46], also in the presence of very severe stress concentration phenomena, the formation of micro/meso

crack goes through an initial Stage I process which is shear dominated, followed by a Stage II propagation that is Mode I governed. With respect to the particular material considered in Figure 8, it is worth observing here that, by testing U-notched flat specimens characterised by stress concentration factors (calculated with respect to the gross sectional area) ranging in the interval 3.8 to 25 [44], the experimental average value of the angle between Stage I planes and notch bisector was seen to be equal to approximately 25° , that is, lower than the value of 45° which would be calculated by determining, according to continuum mechanics, the orientation of the plane experiencing the maximum shear stress/strain amplitude in a homogenous and isotropic material. This discrepancy can simply be ascribed to the fact that, according to Kanazawa, Miller and Brown [21], the orientation of Stage I cracks mainly depends on the actual crystallographic features of the grains in the vicinity of the stress concentrator apices, since Stage I planes in real engineering materials *“are not necessarily the planes of maximum shear in the macroscopic sense, but rather the slip system most closely aligned to these planes”*.

To conclude, it is possible to point out that, since the reasoning summarised in the previous paragraphs does not allow any reasonable hypothesis on the actual shape of the structural volume to be formed, the irregular area sketched in Figure 7 is nothing but a schematic representation of the structural volume idea.

6. IN-FIELD USAGE OF THE PROPOSED APPROACH

In order to use the approach formalised in the previous sections, two different problems have to be addressed explicitly, i.e., (i) the experimental determination of critical distance L_γ , and (ii) the estimation of fatigue lifetime of notched structural components subjected to fatigue loading.

As to the first problem, L_γ can directly be estimated from some calibration tests run using standard notched samples. In particular, by using the PM argument and according to Figure 9a, consider a notched specimen subjected to cyclic axial loading and failing at a number of cycles to failure equal to $\bar{N}_f$. Through appropriate tools (either analytical or numerical) the elasto-plastic stress/strain field

along the focus path can be determined directly, where the relevant stress/strain distributions have to be expressed in terms of both $\sigma_{n,max}$, τ_a , ρ , and γ_a (Fig. 9b). In particular, under complex loading paths, the relevant elasto-plastic stress/strain fields can directly be calculated through commercial Finite Element software packages, provided that, not only the transient behaviour, but also the non-proportional hardening are correctly taken into account [29-32]. The focus path instead is a straight line emanating from the assumed crack initiation point and perpendicular to the surface at the hot-spot itself [1]. For the particular case of standard geometrical features loaded in tension-compression or bending, as, for instance, the one shown in Figure 9b, it is evident that the focus path coincides with the notch bisector.

If the constants in the MMCCM's calibration functions, see Eqs (4) and (5), are known from the experiments for the material being assessed, it is relatively easy to determine the distance from the notch tip, $L_\gamma/2$, at which quantities $\gamma_a = \bar{\gamma}_a$ and $\rho = \bar{\rho}$ (see Fig. 9b) assure the following condition:

$$\bar{\gamma}_a = \frac{\tau'_f(\bar{\rho})}{G} (2\bar{N}_f)^{b(\bar{\rho})} + \gamma'_f(\bar{\rho}) \cdot (2\bar{N}_f)^{c(\bar{\rho})} \quad (17)$$

In other words, $L_\gamma/2$ is nothing but the distance at which the stress/strain state would result, according to the MMCCM, in a fatigue breakage of the plain material at $N_f = \bar{N}_f$ cycles to failure. To conclude, it is not superfluous to highlight that, due to the physiological scattering always characterising fatigue data, to determine L_γ it is always advisable to use several experimental results, possibly generated by testing notches characterised by different sharpness.

As soon as critical distance L_γ is known for the material being investigated, the proposed approach can directly be used to perform the fatigue assessment of mechanical components. In particular, Figure 10 summarises how to use the assessment technique formalised in the present paper to design real structural components against fatigue. In more detail, initially the elasto-plastic stress/strain distributions have to be determined, where the origin of the focus path is taken

coincident with the assumed crack initiation location (point A in Figure 10a). By post-processing the strain state (Fig. 10b) calculated at a distance from the hot-spot equal to $L_\gamma/2$ (point O in Figure 10a), not only the orientation of the critical plane but also γ_a can directly be determined by taking full advantage of one of the available definitions [47] (Fig. 10c). As soon as the critical plane orientation is known, from the stabilised stress state at point O (Fig. 10d) both the shear stress amplitude and the maximum normal stress relative to the critical plane can be calculated directly (Fig. 10e), by also obtaining the corresponding value of ratio ρ , Eq. (2). By taking full advantage of the fully-reversed uniaxial and torsional Manson-Coffin curves as well, the obtained ρ ratio allows the constants in the MMCCM's governing equations – Eqs. (4) and (5) – to be determined (Fig. 10f). Finally, the shear strain amplitude relative to the critical plane, γ_a , together with the appropriate modified Manson-Coffin curve can directly be used to estimate the number of cycles to failure (Fig. 10g).

7. VALIDATION BY EXPERIMENTAL DATA

In order to check the accuracy of the approach formalised in the present paper, two consistent datasets generated by testing notched samples made of aluminium alloy Al6082 [14] and Vanadium-based AISI 1141 MA forging steel [11], respectively, were taken from the technical literature, the static and fatigue properties of the above two materials being listed in Table 1. In more detail, the cylindrical V-notched specimens of Al6082 [9] had gross diameter equal to 10 mm, net diameter equal to 6.1–6.2 mm, notch opening angle equal to 60° , and notch root radii, r , equal to 0.44 mm, 0.5 mm, 1.25 mm, and 4.0 mm, respectively, resulting in linear-elastic stress concentration factors, calculated with respect to the net cross sectional area, K_t , ranging in the interval 1.33 to 2.94. All the above samples were tested under axial force control at R ratios of -1 and 0. As to the AISI 1141 specimens, two different geometrical configurations were considered, i.e., cylindrical bars with circumferential V-notches and flat samples with lateral U-notches. In

more detail, in the cylindrical specimens two values of the notch root radius were investigated, i.e., 0.529 mm and 1.588 mm, and such samples were tested under uniaxial loading at a nominal load ratio R equal to -1. The two flat U-notched geometries instead had notch root radius equal to 2.778 mm and to 9.128 mm, respectively, and they were subjected to cyclic axial loadings characterised by a load ratio equal to zero. In both types of samples, i.e., both bars and plates, the two considered notch root radii resulted in a net K_t value equal to 2.8 (sharp) and to 1.8 (blunt), respectively.

The elasto-plastic stress/strain distributions along the focus paths were determined for all the considered experimental results from elasto-plastic Finite Element analyses done using commercial Finite Element (FE) software ANSYS. Such numerical solutions were obtained by adopting a multilinear isotropic hardening rule as material constitutive law. In order to correctly determine the stress state at the critical points, 10 virtual cycles were run at any stress level in order to allow the material in the vicinity of the notch tip to reach a stabilized configuration, so that, the maximum and minimum values of the stresses and strains at a distance from the notch tip equal to $L_\gamma/2$ were always determined by post-processing the results of the last virtual cycle. The cylindrical samples were modelled through simple axisymmetric bi-dimensional geometries. On the contrary, since Fatemi and co-workers specifically designed their flat geometries in order to have a plane stress distribution in the vicinity of the notch tips [11], the relevant stress/strain fields in such plates were determined through bidimensional geometries solved under the plane stress hypothesis. Finally, in all the FE models used to determine the necessary stabilised elasto-plastic stress/strain fields, convergence was reached by gradually refining the mapped mesh.

The experimental, N_f , vs. estimated, $N_{f,e}$, fatigue lifetime diagrams reported in Figure 11 show the accuracy of the proposed approach in estimating the considered experimental results. In particular, for any investigated material, two results generated, under fully-reversed nominal loading, by testing the sharpest geometrical feature were used to determine, according to the procedure summarised in the previous section, the necessary critical distance value, obtaining a L_γ value of 0.298 mm for aluminium alloy Al6082 and of 0.152 mm for vanadium-based AISI 1141. As to the

MMCCM's accuracy in estimating the considered notch fatigue results, it is worth observing that, according to the values of ratio ρ reported in the two charts of Figure 11, all the post-processed elasto-plastic stress/stain field were characterised, at the critical points, by a $\sigma_{n,max}$ to τ_a ratio larger than unity: this simply means that our approach was capable of simultaneously taking into account not only the notch, but also the mean stress effect by simply treating them as phenomena resulting from inherent multiaxiality.

Lastly, the chart reported in Figure 11a shows that, as far as notched specimens of Al6082 are concerned, our approach was capable of estimates falling within an error factor of about 3, that is, within a scatter band as wide as the one containing the plain results used to determine the appropriate uniaxial Manson-Coffin curve (whose constants are listed in Table 1). This result is very encouraging, since we cannot ask a predictive method to be, from a statistical point of view, more accurate than the experimental information used to calibrate the method itself. On the contrary, the diagram of Figure 11b shows that, since the vanadium-based AISI 1141 was clearly characterised by a lower intrinsic scattering, the use of our approach resulted in estimated falling within an error factor of about 2.

It is possible to conclude the present validation exercise by observing that the high accuracy level shown by the charts of Figure 11 seems to fully support the validity of all the hypotheses formed in the previous Sections and adopted to formalise the fatigue life estimation technique proposed in the present paper.

8. CONCLUSIONS

1) The promising accuracy level shown by the proposed multiaxial elasto-plastic approach seems to strongly support the idea that both the mean stress and notch effect in fatigue can efficiently be treated as problems of elasto-plastic inherent multiaxiality.

- 2) Under uniaxial fatigue loading, the use of the MMCCM in the presence of superimposed static loading is seen to result in estimates as accurate as those obtained by applying both the SWT parameter and Morrow's corrections;
- 3) More work needs to be done in this area to check the validity of such a *modus operandi* also in the presence of sharp notches subjected to complex systems of external constant and variable amplitude cyclic forces, by also considering in full the non-proportional hardening effect.

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Tables

Material	Inconel 718	SAE 1045	En3B ^a	Al 6082 ^a	AISI 1141 ^a
Ref.	[27]	[26]	[14]	[14]	[11]
σ_{UTS} [MPa]	1420	621	622	367	875
σ_y [MPa]	1160	380	571	347	524
E [MPa]	208500	204000	208500	69090	200000
G [MPa]	77800	80300	80000	26000	77000
ϵ'_f	2.67	0.298	0.2113	30.8	1.0266
σ'_f [MPa]	1640	930	691	513.3	1296
b	-0.06	-0.106	-0.0795	-0.0574	-0.08855
c	-0.82	-0.49	-0.4859	-1.4864	-0.6868
γ'_f	18	0.413	0.366	53.4	1.7781
τ'_f [MPa]	2146	505	399	296.3	748
b₀	-0.148	-0.097	-0.0795	-0.0574	-0.08855
c₀	-0.922	-0.445	-0.4859	-1.4864	-0.6868
K' [MPa]	1530	1258	890.7	401.5	1205
n'	0.07	0.208	0.1635	0.024	0.122

^aTorsional fatigue constants estimated according to von Mises.

Table 1: Static and fatigue properties of the investigated materials.

Figures

Figure 1: Adopted fatigue damage model.

(a)

(b)

Figure 2: Modified Manson-Coffin Diagram (a) and accuracy of the MMCCM in estimating lifetime under external multiaxial fatigue loading (b) (for the data sources see the references listed in [5]).

Figure 3: Stress/strain quantities relative to the critical plane under uniaxial fatigue loading under a strain (a) and load ratio (b) larger than -1.

(a)

(b)

Figure 4: Dependency of stress ratio ρ from both the load ratio, R , and the out-of-phase angle, δ in purely linear-elastic materials [1].

Figure 5: MMCCM's accuracy in estimating the mean stress effect under uniaxial fatigue loading.

(a)

(b)

Figure 6: MMCCM vs. classical strain (a) and stress based formulas (b) in predicting the detrimental effect of non-zero mean stresses.

Figure 7: Structural Volume, TCD and MMCCM.

Figure 8: Medium/high cycle fatigue Stage I and Stage II cracks in U-notched samples of low-carbon steel C10 subjected to zero-tension uniaxial loading [37].

Figure 9: Experimental procedure for the determination of critical distance L_γ .

Figure 10: In-field use of the proposed approach.

(a)

(b)

Figure 11: Proposed approach's accuracy in simultaneously estimating both the notch and the mean stress effect under uniaxial fatigue loading.